**Correlation & Regression**

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|  | **Correlation Concept** |  |

Consider the data from five subjects that were asked miles and minutes to arrive at a destination.

Let X=Miles and Y=Minutes.

**Does there seem to be a relationship between the two variables?**

|  |  |  |  |
| --- | --- | --- | --- |
| Subject | X | Y |  |
| 1 | 1 | 4 |
| 2 | 3 | 6 |
| 3 | 3 | 20 |
| 4 | 5 | 15 |
| 5 | 8 | 20 |

Discussion:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1. If X and Y increase (or decrease) together, then there is a positive relationship between X and Y. However, if X increases as Y decreases (or X decreases as Y increases), then there is a negative relationship2. To represent the relationship between the two variables, consider the sum of squared differences between the two variables, SSXY = ( X –X )\*( Y –Y ) 3. If X and Y increase (or decrease) together, then SSXY>0 and there is a positive relationship.4. However, if X increases as Y decreases (or X decreases as Y increases), then SSXY<0 and there is a negative relationship.5. Examples:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Index | X | Y | SSXY |  |
| 1 | 2 | 1 | (2-4)\*(1-3) = +4 |
| 2 | 4 | 3 | (4-4)\*(3-3) = 0 |
| 3 | 6 | 5 | (6-4)\*(5-3) = +4 |
|  |  |  |  SSXY = Sum = +8 |
|  |  |  |  |  |
| Index | X | Y | SSXY |  |
| 1 | 2 | 5 | (2-4)\*(5-3) = –4 |
| 2 | 4 | 3 | (4-4)\*(3-3) = 0 |
| 3 | 6 | 1 | (6-4)\*(1-3) = –4 |
|  |  |  |  SSXY = Sum = –8 |

6. Consider the sample covariance, **Cov(X,Y) = SSXY/(n–1)**7. When SSXY is compared with SSXX and SSYY, it can be shown that (SSXY)2≤(SSXX)\*( SSYY). Therefore, Correlation is defined: **Correlation, r=SSXY/sqrt(SSXX\*SSYY), where (–1 ≤ r ≤ +1).**8. Covariance can be considered an absolute measure of a linear relationship. Whereas, correlation can be considered a relative measure of a linear relationship. . . . |

Example.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Subject | 1 | 2 | 3 | 4 | 5 | Sum | SS |  |  |
| X=Miles | 1 | 3 | 3 | 5 | 8 | 20 |  |  |  |
| Y=Minutes | 4 | 6 | 20 | 15 | 20 | 65 |  |  | SS=Sum of Squares |
| X\*X | 1 | 9 | 9 | 25 | 64 | 108 | 28 |  | SSXX = 108–20\*20/5 = 28SSYY = 1077–65\*65/5 = 232SSXY = 317–20\*65/5 = 57 |
| Y\*Y | 16 | 36 | 400 | 225 | 400 | 1077 | 232 |  |
| X\*Y | 4 | 18 | 60 | 75 | 160 | 317 | 57 |  |
| Correlation, r = SSXY/sqrt(SSXX\*SSYY) = 57/sqrt(28\*232) ≈ 0.707215625 ≈ 0.7072.  |

{ Examples }

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|  | **Correlation Matrix Representations** |  |

Consider three sets of data, A, B, & C, along with their univariate and bivariate measures.

Univariate Measures:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Set | 1 | 2 | 3 | 4 | 5 | Sum = X | X2 | SS = X2 – (X)2/n | S2=SS/(n–1) | S2=SS/n |
| A | 0 | 2 | 1 | 2 | 1 | 6 | 10 | 2.8 | 0.70 | 0.56 |
| B | 4 | 1 | 2 | 0 | 2 | 9 | 25 | 8.8 | 2.20 | 1.76 |
| C | 2 | 0 | 0 | 2 | 0 | 4 | 8 | 4.8 | 1.20 | 0.96 |

Bivariate Measures:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  | Sum(XY) | SSXY | Population Variance/Population Covariance | Correlation, r |
| Set | 1 | 2 | 3 | 4 | 5 | A | B | C | A | B | C | A | B | C | A | B | C |
| A | 0 | 2 | 1 | 2 | 1 | 10 | 6 | 4 | 2.8 | –4.8 | –0.8 | 0.56 | –0.96 | –0.16 |  | –0.967 | –0.218 |
| B | 4 | 1 | 2 | 0 | 2 |  | 25 | 8 |  | 8.8 | 0.8 |  | 1.76 | 0.16 |  |  | 0.123 |
| C | 2 | 0 | 0 | 2 | 0 |  |  | 8 |  |  | 4.8 |  |  | 0.96 |  |  |  |

Matrix Representation of Population Variance, Population Covariance, and Correlation.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Correlation Matrix** |  | **Covariance Matrix** |  | **Variance Matrix** |
|  | A | B | C |  |  | A | B | C |  |  | A | B | C |
| A |  | –0.967 | –0.218 |  | A |  | –0.96 | –0.16 |  | A | 0.56 |  |  |
| B |  |  | 0.123 |  | B |  |  | 0.16 |  | B |  | 1.76 |  |
| C |  |  |  |  | C |  |  |  |  | C |  |  | 0.96 |

Combined Matrix Representation

|  |
| --- |
| **Variance/Covariance/Correlation Matrix** |
|  | **A** | **B** | **C** |
| **A** | 0.56 | –0.96 | –0.16 |
| **B** | –0.967 | 1.76 | 0.16 |
| **C** | –0.218 | 0.123 | 0.96 |

Test of Hypothesis for Significant Correlations

(i.e., Population Correlation, r, is Statistically Significantly Different from Zero.)

|  |
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| **Consider the pair of Hypotheses about the Population Correlation, r**  |
| Ho: r = 0Ha: r ≠ 0 |

Common Representations

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Correlation Matrix** |  | **t-Test Statistic** |  | **p-value** |
|  | A | B | C |  |  | A | B | C |  |  | A | B | C |
| A |  | –0.967 | –0.218 |  | A |  | 6.573\* | 0.387\*\* |  | A |  | 0.007 | 0.724 |
| B |  |  | +0.123 |  | B |  |  | 0.215\*\* |  | B |  |  | 0.844 |
| C |  |  |  |  | C |  |  |  |  | C |  |  |  |
| [ \*(P<0.10), \*\*(P>0.10), for Ho:=0 vs. Ha:r≠0 ] |

{ Examples }