**Introduction to Risk & Utility**

**Risk= [Probability & Outcome] or [Likelihood & Consequence]**

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| --- | --- | --- |
| **Levels of Risk** | low consequence | high consequence |
| high probability | moderate risk | high risk |
| low probability | low risk | moderate risk |

**Utility= Attitude toward risk. (Risk-neutral, Risk-averse, Risk-prone)**

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| Risk Prone |  | Stakeholders |
| Risk Neutral | 🡨can apply to🡪 | Project Team |
| Risk Averse |  | Organization |

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| **Terminology** | | |
| **Measured Risk** | **vs** | **Perceived Risk** |
| ***Risks*** | ***vs*** | ***Barriers*** |
| ***Risk Management*** | ***vs*** | ***Crisis Management*** |

***Consider the ‘Reference Lottery’***

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|  |  |  |  |  |  | |  |  |
|  |  |  |  | (0.5) | Win | | $1 |  |
|  | Risky Decision 🡪 | GO | ($0) |  |  | |  |  |
|  |  |  |  | (0.5) | Lose | | –$1 |  |
|  | Certain Decision 🡪 | NOGO |  |  |  | | $0 |  |
|  |  |  |  |  |  | |  |  |
|  | ↑ | | ↑ ↑ | | | ↑ | | |
|  | Decision  Node | | Chance Probability Outcome  Node | | | | | |
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| **Risk Neutral** |
| A risk neutral decision maker would be indifferent between the decisions in the lottery because the expected value of the risky decision is equivalent to the value of the certain decision. Decision makers not indifferent will exhibit a different “risk preference” or “risk utility” such as risk prone or risk averse. |
| **Risk Prone** |
| A risk prone decision maker would favor the risky decision over the certain decision because the risk in the risky decision will add value to the risky decision relative to the certain decision. |
| **Risk Averse** |
| A risk averse decision maker would favor the certain decision over the risky decision because the risk in the risky decision will subtract value from the risky decision relative to the certain decision. |

Since risk in the risky decision is defined by Probability & Outcome, the amount of risk in the risky decision can be caused by varying the probability, outcome, or both.

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|  | **Utility Theory** |  |

In the following reference lotteries, consider the difference between

risk neutral, risk averse and risk prone decision makers.

**A-1**: The risk neutral decision maker is indifferent between the decisions. The less the magnitude of the outcome, an irrational decision maker usually becomes more risk prone. (Aggressive)

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| --- | --- | --- | --- | --- | --- | --- | --- |
| **A-1** |  |  |  |  |  |  |  |
|  |  |  |  | (0.5) | Win | **$1** |  |
| 🡪 | Risky Decision 🡪 | GO |  |  |  |  |  |
|  |  |  |  | (0.5) | Lose | **–$1** |  |
|  | Certain Decision 🡪 | NOGO |  |  |  | **$0** |  |
|  |  |  |  |  |  |  |  |

**A-2**: The risk neutral decision maker is indifferent between the decisions. The greater the magnitude of the outcome, an irrational decision maker usually becomes more risk averse. (Conservative)

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| **A-2** |  |  |  |  |  |  |  |
|  |  |  |  | (0.5) | Win | **$1M** |  |
|  | Risky Decision 🡪 | GO |  |  |  |  |  |
|  |  |  |  | (0.5) | Lose | **–$1M** |  |
| 🡪 | Certain Decision 🡪 | NOGO |  |  |  | **$0** |  |
|  |  |  |  |  |  |  |  |

**A-3**: When faced with only gains, an irrational decision maker usually chooses the certain decision. The risk neutral decision maker would choose the risky decision. In this case, choosing the certain decision is more risk averse. More risk averse implies a Conservative decision maker.

|  |  |  |  |  |  |  |  |
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| **A-3** |  |  |  |  |  |  |  |
|  |  |  |  | (0.5) | Win | **$2300** |  |
|  | Risky Decision 🡪 | GO |  |  |  |  |  |
|  |  |  |  | (0.5) | Lose | **$0** |  |
| 🡪 | Certain Decision 🡪 | NOGO |  |  |  | **$1000** |  |
|  |  |  |  |  |  |  |  |

**A-4**: When faced with only losses, an irrational decision maker usually chooses the risky decision. The risk neutral decision maker would choose the certain decision. In this case, choosing the risky decision is more risk prone. More risk prone implies an Aggressive decision maker.

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| **A-4** |  |  |  |  |  |  |  |
|  |  |  |  | (0.5) | Win | **$0** |  |
| 🡪 | Risky Decision 🡪 | GO |  |  |  |  |  |
|  |  |  |  | (0.5) | Lose | **–$2300** |  |
|  | Certain Decision 🡪 | NOGO |  |  |  | **–$1000** |  |
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| **Risk Prone** |
| A risk prone decision maker, a risk seeker, a speculative decision maker, would attribute positive value to a risky alternative whether real or perceived. |

Consider a “risk prone” decision maker and the following reference lottery.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **B-1** |  |  |  | (0.5) | Win | $10 |  |
|  | Risky Decision 🡪 | GO | ($0) |  |  |  |  |
|  |  |  |  | (0.5) | Lose | –$10 |  |
|  | Certain Decision 🡪 | NOGO |  |  |  | $0 |  |
|  |  |  |  |  |  |  |  |

A risk prone decision maker would prefer the risky decision, GO, over the certainty decision, NOGO, because, in the judgment of the risk prone decision maker, the expected value of the risky decision will have a greater value than the value of the certainty decision due to the risk present.

One approach to measure the risk attitude is to obtain probabilities such that the risk prone decision maker is indifferent between the two decision alternatives. For example, after a process of soliciting judgment from our risk prone decision maker, suppose the following indifferent reference lottery resulted.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **B-2** |  |  |  | **(0.4)** | Win | $10 |  |
|  | Risky Decision 🡪 | GO | **(-$2)** |  |  |  |  |
|  |  |  |  | **(0.6)** | Lose | –$10 |  |
|  | Certain Decision 🡪 | NOGO |  |  |  | $0 |  |
|  |  |  |  |  |  |  |  |

Since our risk prone decision maker is indifferent between these two decision alternatives, this is stated as “a certain $0 is equivalent in value to a risky -$2” for our risk prone decision maker. In this lottery, the value $0 is called the “certainty equivalent” and the difference between the values of the two decision alternatives, ( -$2 ) - ( $0 ) = -$2, is called the “risk premium”.

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| **Risk Averse** |
| A risk averse decision maker, a risk avoider, a conservative decision maker, would attribute negative value to a risky alternative whether real or perceived. |

Consider a “risk averse” decision maker and the following reference lottery.

|  |  |  |  |  |  |  |  |
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| **C-1** |  |  |  | (0.5) | Win | $10 |  |
|  | Risky Decision 🡪 | GO | ($0) |  |  |  |  |
|  |  |  |  | (0.5) | Lose | –$10 |  |
|  | Certain Decision 🡪 | NOGO |  |  |  | $0 |  |
|  |  |  |  |  |  |  |  |

A risk averse decision maker would prefer the certainty decision, NOGO, over the risky decision, GO, because, in the judgment of the risk averse decision maker, the expected value of the risky decision will have a less value than the value of the certainty decision due to the risk present.

One approach to measure the risk attitude is to obtain probabilities such that the risk averse decision maker is indifferent between the two decision alternatives. For example, after a process of soliciting judgment from our risk averse decision maker, suppose the following indifferent reference lottery resulted.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **C-2** |  |  |  | **(0.6)** | Win | $10 |  |
|  | Risky Decision 🡪 | GO | **(+$2)** |  |  |  |  |
|  |  |  |  | **(0.4)** | Lose | –$10 |  |
|  | Certain Decision 🡪 | NOGO |  |  |  | $0 |  |
|  |  |  |  |  |  |  |  |

Since our risk averse decision maker is indifferent between these two decision alternatives, this is stated as “a certain $0 is equivalent in value to a risky +$2” for our risk averse decision maker. In this lottery, the value $0 is called the “certainty equivalent” and the difference between the values of the two decision alternatives, ( +$2 ) - ( $0 ) = +$2, is called the “risk premium”.

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| **Risk Utility Boundaries** |
| Consider reference lotteries with no risk to establish boundaries. |

To create boundary reference values for risk utility in this lottery, note that if P[Win]=1, then no risk occurs and the indifference lottery would result.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **D-1** |  |  |  | **(1)** | Win | $10 |  |
|  | Risky Decision 🡪 | GO | **($10)** |  |  |  |  |
|  |  |  |  | **(0)** | Lose | –$10 |  |
|  | Certain Decision 🡪 | NOGO |  |  |  | **$10** |  |
|  |  |  |  |  |  |  |  |

Similarly, if P[Win]=0, then no risk occurs and the indifference lottery would result.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **D-2** |  |  |  | **(0)** | Win | $10 |  |
|  | Risky Decision 🡪 | GO | **(-$10)** |  |  |  |  |
|  |  |  |  | **(1)** | Lose | –$10 |  |
|  | Certain Decision 🡪 | NOGO |  |  |  | **-$10** |  |
|  |  |  |  |  |  |  |  |

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|  | **Utility Curves** |  |

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| **Utility Curves from Solicited Information** |
| Using reference lotteries, the “certainty” dollars can be compared to their equivalent “risky” dollars for the different risk utility type decision makers. |

Plot the “Certainty dollars” and “Equivalent Risky dollars” from the results of the solicited reference lotteries.



If the scale of the “Risky dollars” is changed from 0 to 1, then the scale is called a utility scale and the values are called “Utiles”.



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| **Utility Curve Values** |
| Dollars are replaced with utiles from the utility curve indicating the decision makers attitude toward taking risks between -$10 and +$10. |



Consider the utiles obtained from the utility curve.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Monetary Values | -10 | -4 | -3 | 0 | 2 | 4 | 5 | 10 |
| Averse Utiles | 0 | 0.37 | 0.43 | 0.6 | 0.69 | 0.77 | 0.81 | 1 |
| Neutral Utiles | 0 | 0.3 | 0.35 | 0.5 | 0.6 | 0.7 | 0.75 | 1 |
| Prone Utiles | 0 | 0.23 | 0.27 | 0.4 | 0.51 | 0.63 | 0.69 | 1 |

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| **Utility Analysis** |
| To perform risk analysis with utilities, dollars are replaced with utiles from the utility curve in the decision lotteries indicating the decision makers attitude toward taking risks between -$10 and +$10. |

Consider the following examples where dollars are replaced with utiles in brackets.

Consider the utiles obtained from the utility curve.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Monetary Values | -10 | -4 | -3 | 0 | 2 | 4 | 5 | 10 |
| Risk Averse Utiles | 0 | 0.37 | 0.43 | 0.6 | 0.69 | 0.77 | 0.81 | 1 |
| Risk Neutral Utiles | 0 | 0.3 | 0.35 | 0.5 | 0.60 | 0.7 | 0.75 | 1 |
| Risk Prone Utiles | 0 | 0.23 | 0.27 | 0.4 | 0.51 | 0.63 | 0.69 | 1 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **E-1** |  |  | Value | Prob |  | Utiles | $ |  |
|  | **Risk Averse** |  |  | (0.7) | Win | [0.81] | $5 |  |
|  | Risky Decision: | GO | [0.678] |  |  |  |  |  |
|  |  |  |  | (0.3) | Lose | [0.37] | –$4 |  |
| **Decision🡪** | Certain Decision: | NOGO | [0.69] |  |  | [0.69] | $2 |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | **Risk Neutral** |  |  | (0.7) | Win | [0.75] | $5 |  |
| **Decision🡪** | Risky Decision: | GO | [0.615] |  |  |  |  |  |
|  |  |  |  | (0.3) | Lose | [0.30] | –$4 |  |
|  | Certain Decision: | NOGO | [0.60] |  |  | [0.60] | $2 |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | **Risk Prone** |  |  | (0.7) | Win | [0.69] | $5 |  |
| **Decision🡪** | Risky Decision: | GO | [0.552] |  |  |  |  |  |
|  |  |  |  | (0.3) | Lose | [0.23] | –$4 |  |
|  | Certain Decision: | NOGO | [0.51] |  |  | [0.51] | $2 |  |
|  |  |  |  |  |  |  |  |  |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **E-2** |  |  | Value | Prob |  | Utiles | $ |  |
|  | **Risk Averse** |  |  | (0.7) | Win | [0.77] | $4 |  |
|  | Risky Decision: | GO | [0.668] |  |  |  |  |  |
|  |  |  |  | (0.3) | Lose | [0.43] | –$3 |  |
| **Decision🡪** | Certain Decision: | NOGO | [0.69] |  |  | [0.69] | $2 |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | **Risk Neutral** |  |  | (0.7) | Win | [0.70] | $4 |  |
|  | Risky Decision: | GO | [0.595] |  |  |  |  |  |
|  |  |  |  | (0.3) | Lose | [0.35] | –$3 |  |
| **Decision🡪** | Certain Decision: | NOGO | [0.60] |  |  | [0.60] | $2 |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | **Risk Prone** |  |  | (0.7) | Win | [0.63] | $4 |  |
| **Decision🡪** | Risky Decision: | GO | [0.522] |  |  |  |  |  |
|  |  |  |  | (0.3) | Lose | [0.27] | –$3 |  |
|  | Certain Decision: | NOGO | [0.51] |  |  | [0.51] | $2 |  |
|  |  |  |  |  |  |  |  |  |

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| **Utility Curves** |
| Reference Lotteries  Utility Functions  Empirical Data |

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| **Reference Lotteries** | | |
| To determine the utility curve using reference lotteries is merely the process of posing multiple lotteries to the decision maker and eliciting the certainty equivalent, CE, for each risky decision. Then plot values to form the utility curve. | | |
|  | 50-50 Reference Lotteries. This classic approach is to construct a series of reference lotteries containing equal chance probabilities and elicit the CE for different outcome values. |  |
|  | Probability Equivalent, PE, Reference Lotteries. Another approach is to construct a series of reference lotteries containing CE=0 and elicit probabilities that result in an indifference lottery for different outcome values. |  |

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| **Utility Functions** | | |
| A function is chosen to represent the utility curve and the parameters are chosen to reflect the decision maker’s attitude toward risk. Let X=evaluation measure and U=utility measure. Then possible utility functions are: | | |
|  | U=1-exp(-X/R) , R=Risk Tolerance |  |
|  | U=log(X) |  |
|  | U=X^(0.5) |  |
|  | U=X/(X+K) , K=constant |  |

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| **Empirical Data** |
| Estimate a utility curve from observed behavior. A procedure will be presented but mostly the intent of this section is to represent a large class of approaches that will emerge from the logic of utility theory by merely observing the data. |

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|  | **Decision Analysis with Utility Curves** |  |

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| **Decision Analysis with Exponential Utility Curve** |
| Transform the range of payoffs in the Payoff Table to a variable between 0 and 1. Determine utiles from a utility function. Perform decision analysis with probabilities. |

A site for an oil well is under consideration to drill or not to drill. The outcomes defined for the well in this region are dry, small reserve well and large reserve well. Since no information on the probability of the states of reserves is available in this region, a non-informative prior is assumed as a uniform distribution. The payoffs estimated for this region are given in the table.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
|  |  | Reserves | | |  |  |  |
|  | Payoff | Dry | Small | Large | EMV | Decision |  |
|  | Drill | -$2000K | $500K | $3000K | $500K | Yes |  |
|  | NoDrill | 0 | 0 | 0 | 0 |  |  |
|  | Probabilities | 1/3 | 1/3 | 1/3 |  |  |  |
|  |  |  |  |  |  |  |  |

Transform payoffs. Without loss of generality, transform payoffs by “Payoffs/1000”.

From the minimum payoffs of -2000 and maximum payoffs of +3000, define X between 0 and 1 by,

X=(Payoffs – Min)/(Max-Min). Then X=(Payoffs – Min)/(Max-Min)=(Payoffs+2000)/(5000).

Risk Averse Analysis with an exponential utility function and an arbitrary risk tolerance, R=0.5.

The Utiles are determined from U(X)=1-exp(-X/R). The table results.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
|  |  | Payoff($) | Payoff($K) | X | Utiles,U |  |
|  |  | $ | $/1000 | X=( $K – Min)/(Max-Min) | U(X)=1-exp(-X/R), R=0.5 |  |
|  | Min= | -$2000K | -2000 | 0 | 0 |  |
|  |  | $0 | 0 | 0.4 | 0.551 |  |
|  |  | $500K | 500 | 0.5 | 0.632 |  |
|  | Max= | $3000K | 3000 | 1 | 0.865 |  |
|  |  |  |  |  |  |  |

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|  | **Risk Neutral Analysis with transformed payoffs, X** | | | | | |  |
|  |  | Reserves | | |  |  |  |
|  | X | Dry | Small | Large | EMV | Decision |  |
|  | Drill | 0 | 0.5 | 1 | 0.5 | Yes |  |
|  | NoDrill | 0.4 | 0.4 | 0.4 | 0.4 |  |  |
|  | Probability | 0.3333 | 0.3333 | 0.3333 |  |  |  |
|  |  |  |  |  |  |  |  |
|  | **Risk Averse Analysis with Utiles, U(X)=1-exp(-X/R), R=0.5** | | | | | |  |
|  |  | Reserves | | |  |  |  |
|  | Utiles | Dry | Small | Large | EMV | Decision |  |
|  | Drill | 0.0000 | 0.6321 | 0.8647 | 0.4989 |  |  |
|  | NoDrill | 0.5507 | 0.5507 | 0.5507 | 0.5507 | Yes |  |
|  | Probability | 0.3333 | 0.3333 | 0.3333 |  |  |  |
|  |  |  |  |  |  |  |  |

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| --- | --- | --- |
|  | **Decision Analysis with Utility Curves** |  |

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| **Decision Analysis with Exponential Utility Curve** |
| Transform the range of payoffs in the Payoff Table to a variable between 0 and 1.  X=($K-Min)/(Max-Min)  Determine utiles from a utility function. U(X)=1-exp(-X/R), R=Risk Tolerance  Perform decision analysis with probabilities. |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  | **Expected Monetary Value, EMV** | | | | | |  |
|  |  |  |  |  |  |  | Reserves | | |  |  |  |
|  |  |  |  |  |  | Payoff | Dry | Small | Large | EMV | Decision |  |
|  |  |  |  |  |  | Drill | -$2000K | $500K | $3000K | $500K | Yes |  |
|  |  |  |  |  |  | NoDrill | 0 | 0 | 0 | 0 |  |  |
|  |  |  |  |  |  | Probabilities | 1/3 | 1/3 | 1/3 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  | **Risk Neutral Analysis with transformed payoffs, X** | | | | | |  |
|  | $K | X | U,R=0.5 |  |  |  | Reserves | | |  |  |  |
|  | -2000 | 0 | 0 |  |  | X | Dry | Small | Large | EMV | Decision |  |
|  | 0 | 0.4 | 0.551 |  |  | Drill | 0 | 0.5 | 1 | 0.5 | Yes |  |
|  | 500 | 0.5 | 0.632 |  |  | NoDrill | 0.4 | 0.4 | 0.4 | 0.4 |  |  |
|  | 3000 | 1 | 0.865 |  |  | Probability | 0.3333 | 0.3333 | 0.3333 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  | **Risk Averse Analysis with Utiles, U(X)=1-exp(-X/R), R=0.5** | | | | | |  |
|  |  |  |  |  |  |  | Reserves | | |  |  |  |
|  |  |  |  |  |  | Utiles | Dry | Small | Large | EMV | Decision |  |
|  |  |  |  |  |  | Drill | 0.0000 | 0.6321 | 0.8647 | 0.4989 |  |  |
|  |  |  |  |  |  | NoDrill | 0.5507 | 0.5507 | 0.5507 | 0.5507 | Yes |  |
|  |  |  |  |  |  | Probability | 0.3333 | 0.3333 | 0.3333 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

