**Multiple Criteria Decision Making (MCDM)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  | Competing Alternatives |  |
|  | Multiple Criteria | Criteria Weights | Measures |  |
|  |  |  | Weighted Scores |  |
|  | Measures and Criteria Weights are  “Ratio Scale,” “Positive (i.e., >0),” and “higher is better.” | | |  |

Consider

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  | Competing Alternatives | | |  |
|  | Valuable | 🡨&🡪 | Feasible |  |
|  |  |  |  |  |
|  | Multiple Criteria | | |  |
|  | Exhaustive | 🡨&🡪 | Parsimonious |  |
|  |  |  |  |  |
|  | Weights & Measures | | |  |
|  | Accurate | 🡨&🡪 | Precise |  |
|  |  |  |  |  |
|  | Scoring Process | | |  |
|  | Consistent | 🡨&🡪 | Unbiased |  |
|  |  |  |  |  |
|  | Decision | | |  |
|  | Well-informed | 🡨&🡪 | Defendable |  |
|  |  |  |  |  |

**Example: Site Selection**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Two sites are under consideration for a new plant. The decision will be made using two criteria, financial and environmental. The decision committee generated the following judgments. The financial criterion is twice as important as the environmental criterion. After examining all the financial information, site A is considered three times as important as site B. But the environmental information indicated site B is nine times more important than site A. Which site should be chosen?   |  |  |  |  | | --- | --- | --- | --- | |  | **Weight** | **Site A** | **Site B** | | **Financial** | 2 | 3 | 1 | | **Environmental** | 1 | 1 | 9 | | **Weighted Score** |  | **7** | **11** | |  |  | (2\*3+1\*1) **= 7** | (2\*1+1\*9) **= 11** |   Site B is selected (shaded) because Site B Score of 11 is greater than Site A Score of 7.  . . . |

**Examination of Decision**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| . . .  Consider original decision.   |  |  |  |  | | --- | --- | --- | --- | |  | **Weight** | **Site A** | **Site B** | | **Financial** | 2 | 3 | 1 | | **Environmental** | 1 | 1 | 9 | | **Weighted Score** |  | **7** | **11** |   . . .  Honor ratio but increase magnitude of Financial Measures.   |  |  |  |  | | --- | --- | --- | --- | |  | **Weight** | **Site A** | **Site B** | | **Financial** | 2 | **30** | **10** | | **Environmental** | 1 | 1 | 9 | | **Weighted Score** |  | **61** | 29 |   . . . |
| Compare Decisions by examining Bias represented in the Sum of Measures for a Criterion.  Bias due to the magnitude of the measures is called Magnitude Bias. |
| . . .  Consider original decision.   |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | **Weight** | **Site A** | **Site B** | **Sum** | | **Financial** | 2 | 3 | 1 | 4 | | **Environmental** | 1 | 1 | 9 | 10 | | **Weighted Score** |  | **7** | **11** |  |   . . .  Honor ratio but increase magnitude of Financial Measures.   |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | **Weight** | **Site A** | **Site B** | **Sum** | | **Financial** | 2 | **30** | **10** | **40** | | **Environmental** | 1 | 1 | 9 | 10 | | **Weighted Score** |  | **61** | 29 |  |   . . . |

**Obtain unbiased weighted scores by normalizing measures or modifying weights.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Consider original Decision.   |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | Weight | Site A | Site B | Sum | | Financial | 2 | 3 | 1 | 4 | | Environmental | 1 | 1 | 9 | 10 | | Weighted Score |  | 7 | 11 |  |   . . . | |
| **Normalize Measures** | |
| Normalize **measures** for a criterion to eliminate magnitude bias by dividing each measure by the sum of the measures for that criterion. For example, the normalized financial measure for site A is “3/4.” Then, calculate a weighted score using normalized measures to obtain an unbiased weighted score | |
| Eliminate magnitude bias by dividing each measure by the sum of the measures for that criterion.  Then obtain Unbiased Weighted Scores.   |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | Weight | Site A | Site B | Sum | | Financial | 2 | 3/4 | 1/4 | 1 | | Environmental | 1 | 1/10 | 9/10 | 1 | | Unbiased Weighted Score |  | 1.6 | 1.4 | 3.0 | |  |  | 2\*(3/4)+1\*(1/10) | 2\*(1/4)+1\*(9/10) |  |   . . . | |
| **Modify Weights** | |
| Modify **weights** for a criterion to eliminate magnitude bias by dividing each weight by the sum of the measures for that criterion. For example, the modified weight for the financial criterion is “2/4.” Then, calculate a weighted score using modified weights to obtain an unbiased weighted score. | |
| Eliminate magnitude bias by dividing each weight by the sum of the measures for that criterion.  Then obtain Unbiased Weighted Scores using Modified Weights.   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | Weight | Site A | Site B | Sum | Modified  Weights | | Financial | 2 | 3 | 1 | 4 | 2/4 | | Environmental | 1 | 1 | 9 | 10 | 1/10 | | Unbiased Weighted Score |  | 1.6 | 1.4 | 3.0 |  | |  |  | 3\*(2/4)+1\*(1/10) | 1\*(2/4)+9\*(1/10) |  |  |   . . . | |
| **Observations** | |
| 1. The sum of the measures for a criterion indicates the relative amount of bias due to the magnitude of the measures for that criterion.  2. Normalized measures for all criteria will remove decision bias in the weighted scores.  3. Normalized weights have no effect on decision bias in the weighted scores.  4. Sum of the unbiased weighted scores equal the sum of the weights.  5. Measures and weights are “Ratio Scale,” “Positive (i.e., >0),” and “higher is better.” | |

**MCDM Process**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| . . .   |  |  |  | | --- | --- | --- | |  |  | Competing Alternatives | | Multiple Criteria | Criteria Weights | Measures | |  |  | Weighted Scores |   Measures and weights are “Ratio Scale,” “Positive (i.e., >0),” and “higher is better.”  . . . |
| Consider original Decision.   |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | Weight | Site A | Site B | Sum | | Financial | 2 | 3 | 1 | 4 | | Environmental | 1 | 1 | 9 | 10 | | Weighted Score |  | 7 | 11 |  |   . . . |
| **Weights and measures may be obtained the following manner.** |
| Step 1. Obtain Relationships.  Relationships between Criteria. Let F=Financial, E=Environmental.  If F has a weight of 2 and E has a weight of 1, represent this relationship as F=2E.  Measures for Criteria: F=2E. If E=1, then F=2.  Relationships for Criteria between Sites. Let A=Site A, B=Site B.  Measures for Financial: A=3B. If B=1, then A=3.  Measures for Environmental: B=9A. If A=1, then B=9.  . |
| Step 2. Place measures in matrix.   |  |  |  |  | | --- | --- | --- | --- | |  | Weight | Site A | Site B | | Financial | 2 | 3 | 1 | | Environmental | 1 | 1 | 9 | | Weighted Score |  |  |  |   . |
| Step 3. Obtain “Unbiased Weighted Score” using “Normalized Values”.   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | Weight | Site A | Site B | Sum | Modified Weights | | Financial | 2 | 3 | 1 | 4 | 2/4 | | Environmental | 1 | 1 | 9 | 10 | 1/10 | | Unbiased Weighted Score |  | 1.6 | 1.4 | 3.0 |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  |  | Normalized Measures | | |  | Modified Weights | | | |  |  | Weight | Site A | Site B | OR | Weight | Site A | Site B | | Financial |  | 2 | 3/4 | 1/4 |  | 2/4 | 3 | 1 | | Environmental |  | 1 | 1/10 | 9/10 |  | 1/10 | 1 | 9 | | Unbiased Weighted Score |  |  | 1.6 | 1.4 |  |  | 1.6 | 1.4 |   . |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Example1.** “Ratio Scale,” “Positive (i.e., >0),” and “higher is better.”  Three Criteria (C1,C2,C3).  Judgment: C1=4C2, C2=3C3. This implies (=>) C1= 12C3. Thus, if C3=1, then C1=12, C2=3.  Three decisions (D1,D2,D3).  Judgment from C1 information: D1=3D2, D3=4D2. => If D2=1, then D1=3, D3=4.  Judgment from C2 information: D2=D3=0.75D1. => If D1=4, then D2=D3=3.  Judgment from C3 information: D3=D2, D1=3D2. => If D2=D3=1, then D1=3.   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | |  | Weight | D1 | D2 | D3 | Sum | Modified Weights | | C1 | 12 | 3 | 1 | 4 | 8 | 1.5 | | C2 | 3 | 4 | 3 | 3 | 10 | 0.3 | | C3 | 1 | 3 | 1 | 1 | 5 | 0.2 | | Unbiased Weighted Score |  | 6.3 | 2.6 | 7.1 | 16 |  |   . . . |
| **Sensitivity Analysis** |
| 1. Dominated Decisions. “Law of Parsimony”  Since the measures of decision 2 never exceed the measures of the other decisions for all criteria,  Decision 2 is dominated by decision 1 and decision 3.   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | |  | Weight | D1 | D2 | D3 | Sum | Modified Weights | | C1 | 12 | 3 | >1< | 4 | 8 | 1.5 | | C2 | 3 | 4 | >3= | 3 | 10 | 0.3 | | C3 | 1 | 3 | >1= | 1 | 5 | 0.2 |   Since Decision 2 will never be selected, eliminate Decision 2 in the analysis.   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | Weight | D1 | D3 | Sum | Modified Weights | | C1 | 12 | 3 | 4 | 7 | 1.714286 | | C2 | 3 | 4 | 3 | 7 | 0.428571 | | C3 | 1 | 3 | 1 | 4 | 0.25 | |  |  | 7.607143 | 8.392857 | 16 |  |   NOTE: Dominated decisions are “Screened” to be removed from consideration.  Non-dominated decisions are analyzed for “Scoring”, “Ranking” and “Selection”.  . . . |
| 2. Weight Sensitivity analysis. Consider the decision.   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | Weight | D1 | D3 | Sum | Modified Weights | | C1 | W1=12 | 3 | 4 | 7 | 1.714286 | | C2 | W2=3 | 4 | 3 | 7 | 0.428571 | | C3 | W3=1 | 3 | 1 | 4 | 0.25 | |  |  | D1=7.607143 | D3=8.392857 | 16 |  |   Weight sensitivity for this problem will be the limit of each weight that represents the boundary for a decision reversal. Use Solver.  For Criterion, C1: Min D3, By changing W1, subject to D3=Max(D1,D3). Answer, W1=6.5  For Criterion, C2: Min D1, By changing W2, subject to D1=Max(D1,D3). Answer, W2=8.5  For Criterion, C3: Min D1, By changing W3, subject to D1=Max(D1,D3). Answer, W3=2.571428  Range for Weight 1, 6.5 to 12, (12-6.5=5.5)  Range for Weight 2, 3 to 8.5, (8.5-3=5.5)  Range for Weight 3, 1 to 2.571428, (2.571428-1=1.571428)  . . .  NOTE: If the measures for C3 was 1,1 (or any two same numbers) instead of 3,1,  then C3 would provide no distinction between the decisions. C3 can be removed.  . . . |

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| **Example 2**. “Ratio Scale,” “Positive (i.e., >0),” and “higher is better.”  You are evaluating three cars, A,B,C, using the criteria price, performance and prestige. The decision maker establishes price is twice as important as performance. Performance is three times as important as prestige. The prices are $24,000, $36,000 and $48,000 for the cars A,B,C, respectively. The performance measure selected is mileage and the values obtained are 32mpg, 28mpg and 23mpg, for cars A,B,C, respectively. Car B is twice as prestigious as Car A. But Car C is three times as prestigious as Car B. What is the unbiased decision?  . . . |
| Step 1.  Criteria: P1=Price, P2=Performance, P3=Prestige. P1=2P2. P2=3P3. Thus, P3=1, P2=3, P1=6.  Quantitative Price: Measures are: $24,000, $36,000 and $48,000 for the cars A,B,C, respectively  But for Price, lower is better. To transform to higher is better, take reciprocal.  Thus, Measures for Price: 1/24000, 1/36000, 1/48000  Quantitative Performance: Measures are: 32mpg, 28mpg and 23mpg, for cars A,B,C, respectively.  Higher is better. Thus for Performance: 32,28,23  Qualitative Prestige: Measures are B=2A and C=3B. If A=1, then B=2, C=6  . . . |
| Step 2. WLOG, use $K for Price Criterion   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | |  | **Weight** | **CarA** | **CarB** | **CarC** | **Direction** | | **Price** | 6 | 24 | 36 | 48 | Lower is better | | **Performance** | 3 | 32 | 28 | 23 | Higher is better | | **Prestige** | 1 | 1 | 2 | 6 | Higher is better | | **Weighted Score** |  | **241** | **302** | **363** |  |   . . . |
| Step 3. Use reciprocal Price to reverse direction (“Lower is better” to “Higher is better”)   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | |  | **Weight** | **CarA** | **CarB** | **CarC** | **Sum** | **Modified Weights** | | **Price** | 6 | 1/24 | 1/36 | 1/48 | 0.090278 | 66.46154 | | **Performance** | 3 | 32 | 28 | 23 | 83 | 0.03614 | | **Prestige** | 1 | 1 | 2 | 6 | 9 | 0.11111 | | **Unbiased Weighted Score** |  | **4.04** | **3.08** | **2.88** | **10** |  |   . . . |

**Example** 3. Candidates are being considered using weighted criteria.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Criteria | Weights | CandidateA | CandidateB | CandidateC | CandidateD | CandidateE |
|  |  |  |  |  |  |  |
| Experience | 20 | 85 | 80 | 80 | 85 | 90 |
| References | 10 | 90 | 90 | 90 | 70 | 90 |
| Letter | 30 | 60 | 55 | 55 | 30 | 65 |
| Interview | 40 | 80 | 55 | 40 | N/A | 70 |
| Resume | Yes/No | Yes | Yes | Yes | No | Yes |
| Background | Pass/Fail | Pass | Pass | Pass | Fail | Pass |

Step 1. Screening. Eliminate Candidate D due to Background Fail and No Resume

Step 2. Check for Dominated Criteria.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Criteria | CandidateA | CandidateB | CandidateC | CandidateE |  |
|  |  |  |  |  | Max-Min |
| Experience | 85 | 80 | 80 | 90 | 10 |
| References | 90 | 90 | 90 | 90 | 0 |
| Letter | 60 | 55 | 55 | 65 | 10 |
| Interview | 80 | 55 | 40 | 70 | 40 |

Eliminate Criterion “References” due to No Discrimination Capacity, i.e., (Max-Min)=0

Step 3. Check for Dominated Candidates.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Criteria | CandidateA | CandidateB | CandidateC | CandidateE |  |  |  |
|  |  |  |  |  | Max | Min | Max-Min |
| Experience | 85 | 80 | 80 | 90 | 90 | 80 | 10 |
| Letter | 60 | 55 | 55 | 65 | 65 | 55 | 10 |
| Interview | 80 | 55 | 40 | 70 | 80 | 40 | 40 |
| Transform Measures to X=(Measure-Min)/(Max-Min) | | | | | | | |
| Experience | 0.5 | 0 | 0 | 1 |  |  |  |
| Letter | 0.5 | 0 | 0 | 1 |  |  |  |
| Interview | 1 | 0.375 | 0 | 0.75 |  |  |  |

Eliminate Candidate C due to being dominated by other candidates, i.e., X=0 for all criteria

Eliminate Candidate B due to being dominated by other candidates remaining, i.e., X ≤ all criteria

Step 4. Proceed with Unbiased Weighted Scoring

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Criteria | Weights | CandidateA | CandidateE |  |  |
|  |  |  |  | Sum | Modified Weights |
| Experience | 20 | 85 | 90 | 175 | 0.114285714 |
| Letter | 30 | 60 | 65 | 125 | 0.24 |
| Interview | 40 | 80 | 70 | 150 | 0.266666667 |
| Unbiased Weighted Scores | | 45.44761905 | 44.55238095 | 90 |  |
| Decision |  | Yes |  |  |  |

Step 5. Sensitivity Analysis

**Scores**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | Proposal #1 | Proposal #2 | Proposal #3 |
| Criteria | Weight | Scores | Scores | Scores |
| Technical approach | 30 | 72 | 74 | 73 |
| Management approach | 30 | 33 | 29 | 31 |
| Past performance | 20 | 52 | 58 | 60 |
| Price | 20 | 63000 | 47000 | 55000 |
| Weighted Score |  |  |  |  |

**Ranks**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | Proposal #1 | Proposal #2 | Proposal #3 |
| Criteria | Weight | Ranks | Ranks | Ranks |
| Technical approach | 30 | 3 | 1 | 2 |
| Management approach | 30 | 1 | 3 | 2 |
| Past performance | 20 | 3 | 2 | 1 |
| Price | 20 | 3 | 1 | 2 |
| Weighted Score |  |  |  |  |

**Points**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Possible | Proposal #1 | Proposal #2 | Proposal #3 |
| Criteria | Points | Points | Points | Points |
| Technical approach | 30 | 23 | 25 | 24 |
| Management approach | 30 | 22 | 19 | 21 |
| Past performance | 20 | 12 | 13 | 14 |
| Price | 20 | 11 | 15 | 13 |
| Weighted Score |  |  |  |  |

**Unbiased Scores, Ranking, and Points**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | Proposal #1 | Proposal #2 | Proposal #3 | Sum of | Normalized |
| Criteria | Weight | Scores | Scores | Scores | Scores | Weights |
| Technical approach | 30 | 72 | 74 | 73 | 219 | 30/219 |
| Management approach | 30 | 33 | 29 | 31 | 93 | 30/93 |
| Past performance | 20 | 52 | 58 | 60 | 170 | 20/170 |
| Price (Reciprocal) | 20 | 1/63000 | 1/47000 | 1/55000 | K | 20/K |
| Price | 20 | 63000 | 47000 | 55000 |  |  |
|  |  |  |  |  |  |  |
| **Unbiased Weighted Score** |  | **32.36** | **34.01** | **33.63** |  |  |