***Inventory Risk Pooling***

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| Inventory Risk Pooling aggregates inventory through upstream centralized inventory to service multiple downstream demand channels. For the same service levels, inventory risk pooling will usually lower safety stock, lower average inventory, lower inventory carrying cost, and increase efficiency. |

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|  |  | ***Inventory Risk Pooling*** |  |  |
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|  |  | Consider two distinct inventory channel configurations. |  |  |
|  | For the dual channel configuration, the **Dual Stochastic Demand Channels**, X1 and X2, are serviced with two distinct inventories, Inventory-1 and Inventory-2. |  | The **Inventory Risk Pooling** configuration satisfies both stochastic demand channels with one combined inventory. |  |
|  |  | **Dual Stochastic Demand Channels** |  |  |  | **Inventory Risk Pooling** |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | Inventory-1 | X1 |  | Demand-1 |  |  |  |  |  |  | Demand-1 |  |  |
|  |  |  |  |  |  |  |  | X1 |  |  |  |
|  |  |  |  |  |  |  |  |  | Inventory |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | Inventory-2 | X2 |  | Demand-2 |  |  |  |  | X2 |  | Demand-2 |  |  |
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|  | Suppose the two demand channels follow a Normal Distribution, X1~N(1,1) and X2~N(2,2). Suppose each inventory has a 10% stockout level (Z0.10≈1.282) and the same Lead Time (LT). Now consider the safety stock for each inventory in each configuration. |  |
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|  |  | X1 ~ N( 1 , 1 )X2 ~ N( 2 , 2 )SS1 =Z0.10\*1\*sqrt(LT) For Inventory-1SS2 =Z0.10\*2\*sqrt(LT) For Inventory-2 |  |  |  | Let X12 = X1+X2 . Then, X12 ~ N( 1+2 , 12 )SS12=Z0.10\*12\*sqrt(LT) For Combined Inventory |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | Consider a safety stock comparison, SS1+SS2=Z0.10\*(1+2)\*sqrt(LT) SS12=Z0.10\* 12 \*sqrt(LT) |  |  |  |  |
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|  | ***Inventory Risk Pooling*** |  |
|  | 1. It can be shown that 12 ≤ (1+2). Thus, SS12 ≤ SS1+SS2.  |  |
|  | 2. Inventory Risk Pooling will usually result in lower inventory levels and lower inventory carrying cost for the same service level. |  |
|  | 3. The lower the correlation between the demand channels, the greater the cost savings. |  |
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***Inventory Risk Pooling - Analysis***

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Suppose two distinct stochastic demand channels, X1 and X2, are serviced

with two distinct inventories, Inventory-1 and Inventory-2.

The stochastic demand channels follow a Normal Distribution, X1 ~ N( 1 , 1 ) and X2 ~ N( 2 , 2 ).

Risk Pooling satisfies both stochastic demand channels with one combined inventory. Graphically,

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|  |  | **Dual Stochastic Demand Channels** |  |  |  | **Inventory Risk Pooling** |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | Inventory-1 | X1 |  | Demand-1 |  |  |  |  |  |  | Demand-1 |  |  |
|  |  |  |  |  |  |  |  | X1 |  |  |  |
|  |  |  |  |  |  |  |  |  | Inventory |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | Inventory-2 | X2 |  | Demand-2 |  |  |  |  | X2 |  | Demand-2 |  |  |
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Inventory-1 will service a stochastic demand with a mean of 1 and a standard deviation of 1.

Inventory-2 will service a stochastic demand with a mean of 2 and a standard deviation of 2.

The combined inventory in the Inventory Risk Pooling will service a combined stochastic demand with a mean of (1+2) and a standard deviation of 12, which is the mean and standard deviation of (X1+X2).

The inventories contain a statistical safety stock, SS1, SS2 and SS12 for inventory-1, inventory-2 and the combined inventory respectively. Suppose each inventory has a 10% stockout level (Z0.10≈1.282) and the same LT=Lead Time. The statistical safety stock for each inventory is given by

SS1 =Z0.10\*1 \*sqrt(LT) for Inventory-1

SS2 =Z0.10\*2 \*sqrt(LT) for Inventory-2

SS12=Z0.10\*12\*sqrt(LT) for Combined Inventory.

Notice the difference between the safety stocks of the three inventories is the standard deviations.

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| Consider the statistical relationships. |  | Consider the relationship 12 ≤ 1+2) |
| Let X1 ~ N( 1 , 1 )Let X2 ~ N( 2 , 2 )Let 12 = correlation between X1 and X2 .  |  |  The standard deviation of the combined inventory, 12 , will always be less than or equal to the sum of the standard deviations of the distinct inventories, 1+2).  Thus, the safety stock for the combined inventory, SS12 , will always be less than or equal to the sum of the safety stock of the distinct inventories, (SS1+SS2) for the same stockout level and lead time. This implies that multiple stochastic demand channels for the same stockout level and lead time can be serviced with less inventory with a combined inventory.   |
| Var(X1+X2) = Var(X1)+Var(X2)+2\*Cov(X1,X2) (12)2 = (1)2+(2)2+2\*1\*2\*12So, for 12<0, [ Var(X1)+Var(X2) ]>[ Var(X1+X2) ]So, for 12=0, [ Var(X1)+Var(X2) ]=[ Var(X1+X2) ]So, for 12>0, [ Var(X1)+Var(X2) ]<[ Var(X1+X2) ] |  |
| Now,  12 = sqrt[ Var(X1+X2) ] = sqrt[ (1)2+(2)2+2\*1\*2\*12 ]Let 12=1,12 = sqrt[ (1)2+(2)2+2\*1\*2\*1 ] = sqrt[ (1+2)2 ] = (1+2) |  |
| So, for -1<12<+1, 12<1+2) |  | This is called Inventory Risk Pooling. |

Consider the Example.

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| Example  |  |  |  |  |  |  |  |  |
| Index | X1 | X2 |  |  |  | X1+X2 |  |  |
| 1 | 30 | 30 |  |  |  | 60 |  |  |
| 2 | 24 | 20 |  |  |  | 44 |  |  |
| 3 | 35 | 41 |  |  |  | 76 |  |  |
| 4 | 29 | 21 |  |  |  | 50 |  |  |
| 5 | 25 | 35 |  |  |  | 60 |  |  |
| 6 | 33 | 21 |  |  |  | 54 |  |  |
| 7 | 34 | 48 |  |  |  | 82 |  |  |
| 8 | 30 | 30 | Sum |  |  | 60 |  |  |
| Mean | 30 | 30.75 | 60.75 |  | Mean | 60.75 |  |  |
| Variance | 16 | 103.9 | 119.9 |  | Variance | 161.1 |  | The VarianceIncreased 119.9 to 161.1 |
| StandardDeviation | 4 | 10.2 | 14.2 |  | StandardDeviation | 12.7 |  | The Standard DeviationDecreased 14.2 to 12.7 |
| Correlation | 0.50 |  |  |  |  |  |  |

To illustrate the relationship, 12 ≤ 1+2), for -1≤12≤+1,

consider the decrease in standard deviation between 12  and 1+2) for different correlations.

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| Decrease in Standard Deviation | 6.39 | 4.83 | 4.38 | 4.15 | 3.95 | 3.15 | 2.61 | 2.51 | 1.50 | 0.20 |
| Correlation between X1 & X2 | -0.91 | -0.50 | -0.31 | -0.20 | -0.10 | 0.10 | 0.20 | 0.30 | 0.50 | 0.91 |

