***Supply Chain Analytics – Inventory***

Fall 2020

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| **Inventory**. From Simchi-Levi Text, Chapter 2. | |
| ***Outline*** | ***Initial Take-aways*** |
| \*Inventory Model  -Inventory Types  -Inventory Management Plan | \*Supply Chain Inventory Functions and Definitions.  -Inventory Types. Primary, Policy, Supply Chain, Special.  -Inventory Management Plan. Forecasting, Distribution, Evaluation, Sourcing, Policy & Control. |
| \*Inventory Control  -Constant Demand  --Continuous Review (ROP , Q)  --Periodic Review (T , Q)  --Economic Order Quantity (EOQ)  -Stochastic Demand  --Continuous Review (ROP , Q)  --Periodic Review (T , BSL )  --Single Period EOQ | \*Classical inventory control of single-item inventory policy.  -Constant Demand. Fundamental relationships  --Continuous Review (ROP , Q). Define order frequency, inventory turnover rate, safety stock, and stockout level.  --Periodic Review (T , Q). Define inventory period.  --Economic Order Quantity (EOQ). Define minimum cost order quantity.  -Stochastic Demand. Realistic approach to inventory control.  --Continuous Review. Computerized monitoring. For example, low mean high variability inventory demand.  --Periodic Review. Established consistent monitoring. For example, high mean low variability inventory demand.  --Single Period EOQ. Unique inventory policy. For example, rapidly changing product design, variable cost parameters, or long lead times. |
| \*Inventory Risk Pooling | \*Aggregates inventory through upstream centralized inventory to service multiple downstream demand channels. For the same service levels, inventory risk pooling will usually lower safety stock, lower average inventory, lower inventory carrying cost, and increase efficiency. |
| \*Echelon Inventory | \*Addresses inventory control policies for multiple stages within a supply chain. Coordinates and increases efficiency between supply chain stages. |
| \*ABC Classification | \*Practical inventory control of multiple-item inventories. Simple approach, heuristic technique, and effective management of large, co-located or distributed inventories. |

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| **Objectives of Supply Chain Management** |
| Balance “High Service Levels” with “Low Costs”  Emphasize “Continual Improvement” |

***Supply Chain Management – Inventory***

***Supply Chain Inventory Model***

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| *🡨 Flow of Information 🡨*  *🡪 Flow of Material 🡪*   |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | Supplier | | 🡪 | Manufacturer | | 🡪 | Distributor | | 🡪 | Retailer | | 🡪 | Customer | | |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |  | |  |  | |  |  |  |  |  | |  |  | | |  | |  |  | |  |  |  |  |  | |  |  | |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | Policy | 🡨 | Order |  | Demand | 🡨 | Forecasting | | & Control |  |  |  |  |  |  | |  |  |  | Inventory |  |  |  | |  |  |  |  |  |  |  | | Sourcing | 🡪 | Receive |  | Send | 🡪 | Distribution | |  |  |  | **** |  |  |  | |  |  |  | Evaluation |  |  |  |   . . . |

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| ***Inventory Control*** |
| *Determine inventory policies of lot sizing & lot timing*  *that balance the objectives of cost & service levels.* |
| * Inventory Types   + Primary Types 🡪 Raw material, WIP (Work In Process), FGI (Finished Goods Inventory)   + Inventory Policy Types 🡪 Order Quantity, Safety Stock.   + Supply Chain Types 🡪 Pipeline (Logistics), Warehouse (Centralized), Positioned (Decentralized)   + Special Types 🡪 Anticipation (Seasonal), Speculative (Hedge). * Inventory Management Plan   + Forecasting 🡪 *Variability* 🡪 Time Series, Causal, Risk Pooling   + Distribution 🡪 *Service Levels* 🡪 Logistics (Centralized, Distributed, VMI)   + Evaluation 🡪 *Efficiency* 🡪 Turnover Ratio, EPP=(Average inventory)/(Order frequency)   + Sourcing 🡪 *Supply Contracts* 🡪     - (Buy-Back, Revenue-Sharing, Quantity-Flexibility, Sales Rebate)   + Policy & Control 🡪 *Strategy* 🡪     - Inventory Policy: Lot Sizing & Lot Timing     - Inventory Objectives: Service Levels (Stockout Levels) & Cost (EOQ)     - Inventory Period: Single Period & Multiple Periods     - Inventory Item: Single item & Multiple items (ABC Classification)     - Inventory Stage: Single stage & Multiple stages (Echelon)     - Inventory Review: Continuous & Periodic     - Inventory Demand: Deterministic & Stochastic   . . . |

**Inventory Management Plan – Policy & Control**

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|  |  | ***Single Item Summary*** | |  |  |
|  |  | Constant Demand (Deterministic Demand) | |  |  |
|  |  |  |  |  |  |
|  |  | Basic Inventory Policy 🡪 | Q=D\*T |  |  |
|  |  | Reorder Point 🡪 | ROP=D\*LT+SS |  |  |
|  |  | Economic Order Quantity 🡪 | EOQ=sqrt[2\*D\*Co/Cc] |  |  |
|  |  |  |  |  |  |
|  |  | Continuous Review 🡪 | (ROP,Q) |  |  |
|  |  | Periodic Review 🡪 | (T,Q) |  |  |
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|  |  | Stochastic Demand (Probabilistic Demand) | |  |  |
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|  |  | Stochastic Demand 🡪 | Stochastic Demand: Xd ~ N( d , 2d ),  where d = Mean,  2d = Variance, and  d = Standard Deviation |  |  |
|  |  | Order Quantity 🡪 | Q=d \*T |  |  |
|  |  | Stockout Level, , 🡪  Statistical Safety Stock 🡪 | ROP=d\*LT+SS  SSZ \* d \* sqrt(LT) |  |  |
|  |  | Base Stock Level 🡪 | BSL = QBSL + SS  QBSL=(T+LT)\*d  SSZ \* d \* sqrt(T+LT) |  |  |
|  |  | Continuous Review 🡪 | (ROP,Q) |  |  |
|  |  | Periodic Review 🡪 | (T,BSL) 🡨Stochastic Demand  (T,Q) 🡨Deterministic Demand |  |  |
|  |  | EOQ Single Period 🡪 | EOQ = d + Z \* d where =CL/(CL+CS) |  |  |
|  |  | Stochastic Demand & Lead Time 🡪 | Q=d \*T  ROP= d \*L + Z \* sqrt( L\*2d + d\*2L ) |  |  |
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|  |  | Inventory Risk Pooling | |  |  |
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|  |  | Echelon Inventory | |  |  |
|  |  | Echelon Inventory Policy 🡪 | EOQ=sqrt[2\*D\*Co/Cc]  ROPe = d \* LTe + Z \* d \* sqrt( LTe )  Echelon lead time = LTe = LT + downstream LT |  |  |
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|  |  | ***Multiple Items Summary*** | |  |  |
|  |  |  |  |  |  |
|  |  | ABC Inventory Classification | |  |  |
|  |  | Economic ABC Inventory Policy | |  |  |
|  |  | Item Analysis E–ABC Inventory Policy | |  |  |
|  |  | Economically Balanced E–ABC Inventory Policy | |  |  |
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|  | Constant Demand (Deterministic Demand) | |  |
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|  | Basic Inventory Policy 🡪 | Q=D\*T |  |
|  | Reorder Point 🡪 | ROP=D\*LT+SS |  |
|  | Economic Order Quantity 🡪 | EOQ=sqrt[2\*D\*Co/Cc] |  |
|  |  |  |  |
|  | Continuous Review 🡪 | (ROP,Q) |  |
|  | Periodic Review 🡪 | (T,Q) |  |
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|  | **Basic Inventory Policy (ROP,Q) or (T,Q)**  Q=D\*T  ROP=D\*LT+SS |  |
|  | \*Lot Size: Q=D\*T  -Let D=demand=539/week, T=Inventory Period=1.5 weeks  -Then Q=(539)\*(1.5)=808.5809  \*Reorder Point: ROP=D\*LT+SS  - Let Lead Time, LT=0.2 weeks, Safety Stock, SS=10  -Then ROP=D\*LT+SS=(539)\*(0.2)+10=107.8+10118  \*Inventory policy is (ROP,Q) = (118,809) for Continuous Review System.  \*Inventory policy is (T,Q) = (1.5 weeks,809) for Periodic Review System.  **Sawtooth Curve**  Q=809  SS=10  SS=Safety Stock  ROP=118  T=1.5 weeks = Inventory Period  LT=Lead Time=0.2 weeks  . |  |
|  | \*In a Continuous Review System, when the inventory level reaches ROP, order Q=Lot Size.  \*In a Periodic Review System, every inventory period T, order Q=Lot Size.  \*The SS=Safety Stock guards against Stockout. Stockout is unmet demand.  \*Order Frequency is F = orders/time  \*Inventory Turnover = D. Inventory Turnover Rate, TR = Turnover/(Average Inventory)  \*Greater TR implies lower inventory levels, lower investment in inventory, lower risk of obsolescence, greater quality control, greater product control, but greater risk of stockout.  \*Achieving benefits from increasing TR by lowering lot size implies good knowledge of demand, lead time, and unit costs.  . . . |  |

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|  | Constant Demand (Deterministic Demand) | |  |
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|  | Basic Inventory Policy 🡪 | Q=D\*T |  |
|  | Reorder Point 🡪 | ROP=D\*LT+SS |  |
|  | Economic Order Quantity 🡪 | EOQ=sqrt[2\*D\*Co/Cc] |  |
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|  | Continuous Review 🡪 | (ROP,Q) |  |
|  | Periodic Review 🡪 | (T,Q) |  |
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|  | **Basic Inventory Policy (ROP,Q) or (T,Q)**  Q=D\*T  EOQ=sqrt[2\*D\*Co/Cc] |  |
|  | \*   |  |  | | --- | --- | | C = Unit Acquisition Cost | TAC = Total Acquisition Cost =D\*C | | CO = Unit Ordering Cost | TOC = Total Ordering Cost =(order frequency)\*CO | | CC = Unit Carrying Cost | TCC = Total Carrying Cost=(average inventory)\*CC | |  | TIC = Total Inventory Cost =TOC+TCC | |  | TC = Total Cost =TOC+TCC+TAC | | **For SS=0, consider:**  TC = TCC + TOC + TAC  TC = (Ave. Inv.)\*CC + (Order Freq.)\*CO + (Total Items)\*C  TC = ( Q / 2 )\*CC + ( D / Q )\*CO + ( D )\*C  Solving for Q that minimizes TC, Q=EOQ= sqrt( 2 \* D \* CO / CC ). | | |  | |   .   |  |  | | --- | --- | | At EOQ, TCC=TOC | 🡨TIC=TCC+TOC  🡨TCC=(Q/2)\*Cc  🡨TOC=(D/Q)\*Co  . |   . |  |
|  | \*At EOQ for SS=0, TCC=TOC. For Q>EOQ, TCC>TOC. For Q<EOQ, TCC<TOC.  \*Economic Part Period, EPP=(Average Inventory)/(Order Frequency)  -EOQ = sqrt(2\*D\*Co/Cc) 🡪 EOQ\*EOQ=(2\*D)\*(Co/Cc) 🡪 (EOQ/2)\*(EOQ/D)=(Co/Cc)  -EPP = (EOQ/2)/(D/EOQ) = (Average Inventory)/(Order Frequency)  Thus, optimal EPP = Co/Cc = EOQ2/(2\*D) = (Average Inventory)/(Order Frequency)  -EPP based on EOQ yields Optimal EPP.  -(Average Inventory)/(Order Frequency) that deviates from the Optimal EPP will increase total inventory costs.  . . . |  |

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|  | Stochastic Demand (Probabilistic Demand) | |  |
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|  | Stochastic Demand 🡪 | Stochastic Demand: Xd ~ N( d , 2d ),  where d = Mean,  2d = Variance, and  d = Standard Deviation |  |
|  | Order Quantity 🡪 | Q=d \*T |  |
|  | Stockout Level, , 🡪  Statistical Safety Stock 🡪  Reorder Point, ROP 🡪 | ROP=d\*LT+SS  SSZ \* d \* sqrt(LT)  ROP=d \* LT + Z \* d \* sqrt(LT) |  |
|  | Base Stock Level 🡪  Base Stock Level, BSL 🡪 | BSL = QBSL + SS  QBSL=(T+LT)\*d  SSZ \* d \* sqrt(T+LT)  BSL= (T+LT) \*dZ \* d \* sqrt(T+LT) |  |
|  | Continuous Review 🡪 | (ROP,Q) |  |
|  | Periodic Review 🡪 | (T,BSL) 🡨Stochastic Demand  (T,Q) 🡨Deterministic Demand |  |
|  | EOQ Single Period 🡪 | EOQ = d + Z \* d where =CL/(CL+CS) |  |
|  | Stochastic Demand &  Stochastic Lead Time | Q=d \*T  ROP= d \*L + Z \* sqrt( L\*2d + d\*2L ) |  |
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|  | Inventory Risk Pooling | |  |
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**Stochastic Demand**

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|  | **Statistical Safety Stock**  ROP=D\*LT+SS 🡨Deterministic  ROP=d\*LT+SS 🡨Stochastic  SSZ \* d \* sqrt(LT) |  |
|  | Stochastic Demand: Xd ~ N( d , 2d )   |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  | | d X Xd  Xd ~ N( d , 2d ) | | | |   .   |  |  |  | | --- | --- | --- | | **Sawtooth Curve**   |  |  | | --- | --- | | ROP=D\*LT | LT | |   Stochastic Demand during lead time: XLT ~ N( LT\*d , LT\*2d ).   |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  | | LT\*d X XLT  XLT ~ N( LT\*d , LT\*2d ) | | | |   . . . |  |

**Standard Normal Distribution**

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|  | **Statistical Safety Stock**  ROP=D\*LT+SS 🡨Deterministic  ROP=d\*LT+SS 🡨Stochastic  SSZ \* d \* sqrt(LT) |  |
|  | Stochastic Demand: Xd ~ N( d , 2d )   |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  | | d X Xd  Xd ~ N( d , 2d ) | | | |   Stochastic Demand during lead time: XLT ~ N( LT\*d , LT\*2d )   |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  | | LT\*d X XLT  XLT ~ N( LT\*d , LT\*2d ) | | | |   Both Xd and XLT can be transformed to a Standard Normal Variable, Z,  Where Z=(X–)/ for X~N( , 2 ) and Z~N( 0 , 1 )  So, for Xd ~ N( d , 2d ), Z = (Xd)/sqrt(2d)  And, for XLT ~ N( LT\*d , LT\*2d ), Z=(XLT\*d)/sqrt(LT\*2d)   |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  | | 0 Z Z  Z ~ N(  , 1 ) | | | |   . . . |  |

**Stochastic Demand: SSZ \* d \* sqrt(LT)**

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|  | | **Statistical Safety Stock**  Q=d\*T  ROP=d\*LT+SS  SSZ \* d \* sqrt(LT)  Inventory Policy, (ROP,Q)=( d\*LT +Z \* d \* sqrt(LT) , d\*T ) | |  | |
|  | | \* Stochastic Demand: Xd ~ N( d , 2d ),  where d = 539/week and 2d = 6724/week2 . Thus, d = 82/week.  \*Reorder point: ROP= d\*LT + SS  -Stockout is when demand exceed inventory during an inventory period  -Add SS=Safety Stock to ROP to lower risk of Stockout  -Demand during lead time: XLT ~ N( LT\*d , LT\*2d ).  -Stockout Level:  = P[ XLT > LT\*d + SS]  -Statistical Safety Stock: SSZ \* d \* sqrt(LT)  -Let LT=0.2 weeks and  = 0.10, so Z ≈ 1.282  -Then, SS= Z \* d \* sqrt(LT) = 1.282 \* 82 \* sqrt(0.2) ≈ 47  -And, ROP= d\*LT+SS = d\*LT+SS = (539)\*(0.2)+47≈155   |  | | --- | | **Sawtooth Curve**  ROP SS=Safety Stock LT |   \*Inventory policy is (ROP,Q) = (155,809).  -When the inventory level equals 155, order 809. | |  | |
|  | | \*Stockout Level = 1–Service Level  \*As SS increases, Service Level increases and Stockout Level decreases.  \*As SS increases, Inventory costs increase and Stockout costs decrease.  \*Optimal SS is at break-even between inventory costs and stockout costs. | |  | |
|  | | \*Since, **SS**Z\*d\*sqrt(LT), SS will increase as service level increases, demand variability increases, or lead time increases.  \*\*Reducing cost must balance service levels with demand variability and lead times.  -Manage demand variability: risk pooling, forecasting techniques.  -Manage lead times: centralization of information, electronic ordering, information sharing, and VMI (vendor managed inventories). | |  | |
|  | | \*Continuous Review. Computerized monitoring. For example, low mean high variability inventory demand. | |  | |

**Stochastic Demand: Base Stock Level, BSL = QBSL + SS**

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|  | **Base Stock Level, BSL = QBSL + SS**  QBSL=(T+LT)\*d ; SSZ \* d \* sqrt(T+LT) |  |
|  | \*Periodic review system with stochastic demand requires an inventory policy at varying inventory levels at specified inventory periods.  \*Stochastic Demand, Xd ~ N( d , 2d ) ,  where d = 539/week and 2d = 6724/week2 . Thus, d = 82/week  \*Inventory Period, T=1.5 weeks.  \*Lead Time, LT=0.2 weeks  \*Stockout Level,  = 0.10, so Z ≈ 1.282  -Then QBSL=(T+LT)\*d  = (1.5+0.2)\*539 = 916  -Then SS= Z \* d \* sqrt(T+LT) = 1.282 \* 82 \* sqrt(1.5+0.2) = 137  -Base Stock Level, BSL = 916 + 137 = 1053  \*Inventory position = on-hand inventory – backorders + orders not received   |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | BSL=Base Stock Level |  |  |  |  | |  |  | |  | |  |  |  |  |  | |  |  | |  | | Q=Lot Size |  |  |  |  | |  |  | |  | | Inventory Position |  |  |  |  | |  |  | |  | |  |  |  | 🡨LT🡪 |  | | 🡨LT🡪 |  | |  | |  |  |  |  | 🡨T🡪 | | |  | |  | |  |  |  | 🡨T🡪 | | |  |  | |  | |  |  |  | | |  | | |  |  |   \*Inventory policy is to order from inventory position up to BSL=1053 every T=1.5 weeks. |  |
|  | \*Since, Base Stock Level=BSL= **QBSL + SS =** (T+LT)\*d + Z \* d \* sqrt(T+LT),  the BSL increases as mean demand increases, demand variability increases, lead time increases, inventory period increases, and service level increases.  \*Base Stock Level primarily addresses service level not cost.  -A relationship is observed between high service levels and high inventory levels.  -A relationship is observed between high service levels and shorter lead times.  -It is observed that a decreased inventory period can decrease demand uncertainty resulting in increased service levels.  -Demand variability can be managed through risk pooling. |  |
|  | \*Periodic Review. Established consistent monitoring. For example, high mean low variability inventory demand. |  |

***Inventory Risk Pooling***

*Michael D. Harper, Ph.D.*

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| Inventory Risk Pooling aggregates inventory through upstream centralized inventory to service multiple downstream demand channels. For the same service levels, inventory risk pooling will usually lower safety stock, lower average inventory, lower inventory carrying cost, and increase efficiency. |

Consider two distinct inventory channel configurations.

For the dual channel configuration, the **Dual Stochastic Demand Channels**, X1 and X2, are serviced with two distinct inventories, Inventory-1 and Inventory-2.

The **Inventory Risk Pooling** configuration satisfies both stochastic demand channels with one combined inventory.

Suppose the stochastic demand channels follow a Normal Distribution, X1~N(1,1) and X2~N(2,2).

Graphically,

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|  |  | **Dual Stochastic Demand Channels** | | | |  |  |  | **Inventory Risk Pooling** | | | |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | Inventory-1 | X1 |  | Demand-1 |  |  |  |  |  |  | Demand-1 |  |  |
|  |  |  |  |  |  |  |  | X1 |  |  |  |
|  |  |  |  |  |  |  |  |  | Inventory |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | Inventory-2 | X2 |  | Demand-2 |  |  |  |  | X2 |  | Demand-2 |  |  |
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|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | Suppose each inventory has a 10% stockout level (Z0.10≈1.282) and the same Lead Time (LT). Now consider the safety stock for each inventory in each configuration. | | | | | | | | | | | | |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | X1 ~ N( 1 , 1 )  X2 ~ N( 2 , 2 )  SS1 =Z0.10\*1\*sqrt(LT)  For Inventory-1  SS2 =Z0.10\*2\*sqrt(LT)  For Inventory-2 | | | |  |  |  | Let X12 = X1+X2 . Then,  X12 ~ N( 1+2 , 12 )  SS12=Z0.10\*12\*sqrt(LT)  For Combined Inventory | | | |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | Consider a safety stock comparison,  SS1+SS2=Z0.10\*(1+2)\*sqrt(LT)  SS12=Z0.10\* 12 \*sqrt(LT) | | | | | | |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | ***Inventory Risk Pooling*** | | | | | | | | | | | | |  |
|  | 1. It can be shown that 12 ≤ (1+2). Thus, SS12 ≤ SS1+SS2. | | | | | | | | | | | | |  |
|  | 2. Inventory Risk Pooling will usually result in lower inventory levels for the same service level. | | | | | | | | | | | | |  |
|  | 3. Inventory Risk Pooling will usually result in lower inventory carrying cost for the same service level. | | | | | | | | | | | | |  |
|  | 4. The lower the correlation between the demand channels, the greater the cost savings. | | | | | | | | | | | | |  |
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| --- | --- | --- | --- |
|  | Echelon Inventory | |  |
|  | Echelon Inventory Policy 🡪 | EOQ=sqrt[2\*D\*Co/Cc]  ROPe = d \* LTe + Z \* d \* sqrt( LTe )  Echelon lead time = LTe = LT + downstream LT |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Echelon Inventory Policy**   |  |  |  |  |  | | --- | --- | --- | --- | --- | | **Stage 3** | **🡪** | **Stage 2** | **🡪** | **Stage 1** |   ROPe = d \* LTe + Z \* d \* sqrt( LTe )  . . . |  |
|  | \*Echelon inventory = on hand + downstream inventory  \*Echelon inventory position = on hand + upstream orders not received - backorders  \*Echelon lead time = LTe = LT + downstream LT |  |
|  | \*Echelon Inventory Policy. When the echelon inventory position is at or below the echelon reorder point, ROPe=d\*LTe+Z\*d\*sqrt(LTe), order the appropriate lot size, Q. |  |
|  | \*Let demand, Xd , for Stage 1 be stochastic, Xd ~ N( d , 2d ) ,  where d = 539/week and 2d = 6724/week2 . Thus, d = 82/week.  \*Stockout Level, =0.10. Thus, Z≈1.282  \*Let unit ordering cost, Co, and unit carrying cost, Cc, be:  -For Stage 1: Co=$249/order, Cc=$0.41/item/week.  -For Stage 2: Co=$254/order, Cc=$0.32/item/week.  -For Stage 3: Co=$260/order, Cc=$0.29/item/week.  \*Lot Size:  -Lot Size for Stage 1: EOQ= sqrt(2\*539\*249/0.41)=809  -Lot Size for Stage 2: EOQ= sqrt(2\*539\*254/0.32)=925  -Lot Size for Stage 3: EOQ= sqrt(2\*539\*260/0.29)=983  \*Let Lead Time, LT, be:  -Lead Time between Stage 1 and Stage 2: LT1=0.2 weeks  -Lead Time between Stage 2 and Stage 3: LT2=0.5 weeks  -Lead Time between Stage 3 and Supplier: LT3=0.3 weeks  \*Echelon Lead Time: LTe  -Echelon Lead Time for Stage 1: LTe1 = 0.2  -Echelon Lead Time for Stage 2: LTe2 = 0.2 + 0.5 = 0.7  -Echelon Lead Time for Stage 3: LTe3 = 0.2 + 0.5+0.3 = 1.0  \*Echelon Reorder Point: ROPe = d \* LTe + Z \* d \* sqrt( LTe )  -Echelon Reorder Point for Stage 1: ROPe1 = 539\*0.2+1.282\*82\*sqrt(0.2)=155  -Echelon Reorder Point for Stage 2: ROPe2 = 539\*0.7+1.282\*82\*sqrt(0.7)=465  -Echelon Reorder Point for Stage 3: ROPe3 = 539\*1.0+1.282\*82\*sqrt(1.0)=644  \*Echelon Inventory Policy:  -Echelon Inventory Policy for Stage 1: (155,809)  -Echelon Inventory Policy for Stage 2: (465,925)  -Echelon Inventory Policy for Stage 3: (644,983)  . . . |  |

**Stochastic Demand: Single Period EOQ**

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| --- | --- | --- | --- | --- | --- |
|  | | **Single Period EOQ**  EOQ = d + Z \* d | |  | |
|  | | \* Stochastic Demand during single period: D ~ N( d , 2d ),  where d = 539/week and 2d = 6724/week2 . Thus, d = 82/week.  \* CLONG = CL = Cost of inventory remaining after period.  (Lot Size too large. Demand less than Lot Size.)  \* CSHORT = CS = Cost of demand not satisfied during a period.  (Lot Size too small. Demand greater than Lot Size.) | |  | |
|  | | \*Determine Lot Size, Q, such that CS \* P( D > Q ) = CL \* P( D < Q )  -So, CS \* P( D > Q ) = CL ( 1 – P( D > Q ) )  -And, P( D > Q ) = CL / (CL + CS ) | |  | |
|  | | \*Define Stockout Level =  = P( D > Q ) = CL / (CL + CS ) | |  | |
|  | | \*The probability, P( D > Q ) , is same as P( Z > ( Q – d ) / d ) =  , from Normal Distribution.  -Comparing with P( Z > Z ) = we have ( Q – d ) / d = Z  -Solving for Q results in Q = d + Z \* d , or EOQ = d + Z \* d | |  | |
|  | | \*Single Period EOQ: EOQ = d + Z \* d  -Stochastic Demand during single period: D ~ N( d , 2d ),  where d = 539/week and 2d = 6724/week2 . Thus, d = 82/week.  -Short Cost, CS = $27. Long Cost, CL = $3.  -Stockout Level =  = 3/(3+27)=0.10. Service Level = 1 – 0.10 = 0.90. So, Z ≈ 1.282  -Then, EOQ = d + Z \* d = 539 + 1.282 \* sqrt(6724) = 644.124 ≈ 644 | |  | |
|  | | \*Inventory policy is order 644 for one period. | |  | |
|  | | \*Since, EOQ = d + Z \* d ,  EOQ will increase as demand mean increases, service level increases,  or demand variability increases. | |  | |
|  | | \*Single Period EOQ. Unique inventory policy. For example, rapidly changing product design, variable cost parameters, or long lead times. | |  | |
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|  |  | ***Multiple Items Summary*** | |  |  |
|  |  |  |  |  |  |
|  |  | ABC Inventory Classification | |  |  |
|  |  | Economic ABC Inventory Policy | |  |  |
|  |  | Item Analysis E–ABC Inventory Policy | |  |  |
|  |  | Economically Balanced E–ABC Inventory Policy | |  |  |
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| **Multiple Items Inventory Policy** |
| **ABC Inventory Classification**   |  |  |  | | --- | --- | --- | | Class | Annual Usage ($) | Total SKU Items | | A | 75% to 80% | 10% to 20% | | B | 10% to 15% | 20% to 40% | | C | 5% to 10% | 40% to 50% |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | SKU | Annual  Usage ($) | Percent  Usage (%) | Cumulative  Percentage | Class | %Usage | %Items | | 5 | 73728 | 64.7 | 64.7 | A |  |  | | 8 | 18432 | 16.2 | 80.9 | A | 80.9 | 20 | | 1 | 8063 | 7.1 | 88.0 | B |  |  | | 6 | 4478 | 3.9 | 91.9 | B |  |  | | 4 | 2819 | 2.5 | 94.4 | B | 13.5 | 30 | | 9 | 2057 | 1.8 | 96.2 | C |  |  | | 7 | 1515 | 1.3 | 97.5 | C |  |  | | 3 | 1161 | 1.0 | 98.5 | C |  |  | | 10 | 920 | 0.8 | 99.3 | C |  |  | | 2 | 747 | 0.7 | 100.0 | C | 5.6 | 50 | | Total | 113920 | 100.0 |  |  |  |  |     . . . |

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| **ABC Classification Cost Analysis** |
| \*Assume Unit Ordering Cost is $40/order and Unit Carrying Cost is $0.25/$/year.  \*Evaluate the arbitrary inventory policy (10,8,4) that represents the annual order frequency for classes A,B,C, respectively.   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | Class  i | Annual  Usage($) Mi | Number of  SKUs  Ni | Order  Frequency  Fi | Total  Orders  NiFi | Average  Inventory($)  Mi/(2Fi) | | A | 92160 | 2 | 10 | 20 | 4608 | | B | 15360 | 3 | 8 | 24 | 960 | | C | 6400 | 5 | 4 | 20 | 800 | | Total | 113920 | 10 |  | 64 | 6368 |  |  |  |  | | --- | --- | --- | | Unit Costs | $40 | $0.25 | | Total Ordering Cost | $2560 |  | | Total Carrying Cost |  | $1592 |  |  |  | | --- | --- | | Total Inventory Cost | $4152 |   **Objective: Determine order frequency that will minimize total inventory cost.**  . . . |

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| **Economic ABC Inventory Policy** |
| Order frequency per class to minimize cost is:  **Fi = sqrt[ Mi/(2\*Ni\*EPP) ], i=A,B,C. EPP=Co/Cc**  This relationship yields, FA = sqrt[ 92160/(2\*2\*(40/0.25))] = 12  FB = sqrt[ 15360/(2\*3\*(40/0.25))] = 4  FC = sqrt[ 6400/(2\*5\*(40/0.25))] = 2  where EPP = 40/0.25 = 160.   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | Class  i | Annual  Usage($) Mi | Number of  SKUs  Ni | Order  Frequency  Fi | Total  Orders  NiFi | Average  Inventory($)  Mi/(2Fi) | EPP | | A | 92160 | 2 | 12 | 24 | 3840 | 160 | | B | 15360 | 3 | 4 | 12 | 1920 | 160 | | C | 6400 | 5 | 2 | 10 | 1600 | 160 | | Total | 113920 | 10 |  | 46 | 7360 |  |  |  |  |  | | --- | --- | --- | | Unit Costs | $40 | $0.25 | | Total Ordering Cost | $1840 |  | | Total Carrying Cost |  | $1840 |  |  |  | | --- | --- | | Total Inventory Cost | $3680 |   . . . |

**APPENDIX A**

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| **Item Analysis E–ABC Inventory Policy** |
| \*From the E-ABC Policy: Fi = sqrt[ Mi/(2\*Ni\*EPP) ], i=A,B,C. EPP=Co/Cc  \*Let each SKU be its own class. Then an optimal order frequency can be determined for each SKU using: Fi = sqrt[ Mi/(2\*EPP) ], i=SKU. EPP=Co/Cc.   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | SKU | Annual  Usage ($) | Number of  SKUs  Ni | Order  Frequency  Fi | Total  Orders  NiFi | Average  Inventory($)  Mi/(2Fi) | EPP | | 5 | 73728 | 1 | 15.179 | 15.179 | 2428.629 | 160 | | 8 | 18432 | 1 | 7.589 | 7.589 | 1214.315 | 160 | | 1 | 8063 | 1 | 5.020 | 5.020 | 803.144 | 160 | | 6 | 4478 | 1 | 3.741 | 3.741 | 598.532 | 160 | | 4 | 2819 | 1 | 2.968 | 2.968 | 474.889 | 160 | | 9 | 2057 | 1 | 2.535 | 2.535 | 405.660 | 160 | | 7 | 1515 | 1 | 2.176 | 2.176 | 348.138 | 160 | | 3 | 1161 | 1 | 1.905 | 1.905 | 304.762 | 160 | | 10 | 920 | 1 | 1.696 | 1.696 | 271.293 | 160 | | 2 | 747 | 1 | 1.528 | 1.528 | 244.459 | 160 | | Total | 113920 | 10 | 44.337 | 44.337 | 7093.821 |  |  |  |  |  | | --- | --- | --- | | Unit Costs | $40 | $0.25 | | Total Ordering Cost | $1773.5 |  | | Total Carrying Cost |  | $1773.5 |  |  |  | | --- | --- | | Total Inventory Cost | $3546.9 |   - |

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| **Economically Balanced E–ABC Inventory Policy** |
| \*From the E-ABC Policy: Fi = sqrt[ Mi/(2\*Ni\*EPP) ], i=A,B,C. EPP=Co/Cc.  \*Suppose the unit costs are unknown.  \*Perform analysis of Class A items and determine the order frequency, FA.  \*Then, the “Observed EPP” would be “[Ave.Inv.]/[TotalOrders]=“[MA/(2FA)]/[NAFA]” for Class A items.  \*Use this Observed EPP for the order frequencies for Class B and Class C items.  Suppose after an extensive Class A analysis, it was determined that FA=15.  Using FA=15, determine the Observed EPP=[92160/(2\*15)]/[2\*15]=102.4 in the table.  Then, FB = sqrt[ 15360/(2\*3\*102.4) ]=5  And, FC = sqrt[ 6400/(2\*5\*102.4) ]=2.5   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | Class  i | Annual  Usage($) Mi | Number of  SKUs  Ni | Order  Frequency  Fi | Total  Orders  NiFi | Average  Inventory($)  Mi/(2Fi) | EPPO | | **A** | **92160** | **2** | **15** | **30** | **3072** | **102.4** | | B | 15360 | 3 | 5 | 15 | 1536 | 102.4 | | C | 6400 | 5 | 2.5 | 12.5 | 1280 | 102.4 | | Total | 113920 | 10 |  | 57.5 | 5888 |  |   - |
| \*Although not optimal, all classes will have the same Observed EPP. This is called an “Economically Balanced” inventory, EB-ABC Inventory Policy.  \*Although not optimal, an economically balanced inventory will be based on the accuracy of the Class A inventory analysis.  \*The EB-ABC inventory analysis can also be used to obtain an Item EB-ABC policy.  \*It can be shown that the structure of an EB-ABC inventory can be used for further analysis and strategy.  \*\*All ABC Inventory analyses can be used for an Echelon Inventory.  - |

**APPENDIX B**

***Inventory Risk Pooling***

*Michael D. Harper, Ph.D.*

Suppose two distinct stochastic demand channels, X1 and X2, are serviced

with two distinct inventories, Inventory-1 and Inventory-2.

The stochastic demand channels follow a Normal Distribution, X1 ~ N( 1 , 1 ) and X2 ~ N( 2 , 2 ).

Risk Pooling satisfies both stochastic demand channels with one combined inventory. Graphically,

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
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|  |  | **Dual Stochastic Demand Channels** | | | |  |  |  | **Inventory Risk Pooling** | | | |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | Inventory-1 | X1 |  | Demand-1 |  |  |  |  |  |  | Demand-1 |  |  |
|  |  |  |  |  |  |  |  | X1 |  |  |  |
|  |  |  |  |  |  |  |  |  | Inventory |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | Inventory-2 | X2 |  | Demand-2 |  |  |  |  | X2 |  | Demand-2 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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Inventory-1 will service a stochastic demand with a mean of 1 and a standard deviation of 1.

Inventory-2 will service a stochastic demand with a mean of 2 and a standard deviation of 2.

The combined inventory in the Inventory Risk Pooling will service a combined stochastic demand with a mean of (1+2) and a standard deviation of 12, which is the mean and standard deviation of (X1+X2).

The inventories contain a statistical safety stock, SS1, SS2 and SS12 for inventory-1, inventory-2 and the combined inventory respectively. Suppose each inventory has a 10% stockout level (Z0.10≈1.282) and the same LT=Lead Time. The statistical safety stock for each inventory is given by

SS1 =Z0.10\*1 \*sqrt(LT) for Inventory-1

SS2 =Z0.10\*2 \*sqrt(LT) for Inventory-2

SS12=Z0.10\*12\*sqrt(LT) for Combined Inventory.

Notice the difference between the safety stocks of the three inventories is the standard deviations.

|  |  |  |
| --- | --- | --- |
| Consider the statistical relationships. |  | Consider the relationship 12 ≤ 1+2) |
| Let X1 ~ N( 1 , 1 )  Let X2 ~ N( 2 , 2 )  Let 12 = correlation between X1 and X2 . |  | The standard deviation of the combined inventory, 12 , will always be less than or equal to the sum of the standard deviations of the distinct inventories, 1+2).  Thus, the safety stock for the combined inventory, SS12 , will always be less than or equal to the sum of the safety stock of the distinct inventories, (SS1+SS2) for the same stockout level and lead time.  This implies that multiple stochastic demand channels for the same stockout level and lead time can be serviced with less inventory with a combined inventory. |
| Var(X1+X2) = Var(X1)+Var(X2)+2\*Cov(X1,X2)  (12)2 = (1)2+(2)2+2\*1\*2\*12  So, for 12<0, [ Var(X1)+Var(X2) ]>[ Var(X1+X2) ]  So, for 12=0, [ Var(X1)+Var(X2) ]=[ Var(X1+X2) ]  So, for 12>0, [ Var(X1)+Var(X2) ]<[ Var(X1+X2) ] |  |
| Now,  12 = sqrt[ Var(X1+X2) ]  = sqrt[ (1)2+(2)2+2\*1\*2\*12 ]  Let 12=1,  12 = sqrt[ (1)2+(2)2+2\*1\*2\*1 ]  = sqrt[ (1+2)2 ]  = (1+2) |  |
| So, for -1<12<+1, 12<1+2) |  | This is called Inventory Risk Pooling. |

Consider the Example.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Example |  |  |  |  |  |  |  |  |
| Index | X1 | X2 |  |  |  | X1+X2 |  |  |
| 1 | 30 | 30 |  |  |  | 60 |  |  |
| 2 | 24 | 20 |  |  |  | 44 |  |  |
| 3 | 35 | 41 |  |  |  | 76 |  |  |
| 4 | 29 | 21 |  |  |  | 50 |  |  |
| 5 | 25 | 35 |  |  |  | 60 |  |  |
| 6 | 33 | 21 |  |  |  | 54 |  |  |
| 7 | 34 | 48 |  |  |  | 82 |  |  |
| 8 | 30 | 30 | Sum |  |  | 60 |  |  |
| Mean | 30 | 30.75 | 60.75 |  | Mean | 60.75 |  |  |
| Variance | 16 | 103.9 | 119.9 |  | Variance | 161.1 |  | The Variance  Increased 119.9 to 161.1 |
| Standard  Deviation | 4 | 10.2 | 14.2 |  | Standard  Deviation | 12.7 |  | The Standard Deviation  Decreased 14.2 to 12.7 |
| Correlation | 0.50 | |  |  |  |  |  |  |

To illustrate the relationship, 12 ≤ 1+2), for -1≤12≤+1,

consider the decrease in standard deviation between 12  and 1+2) for different correlations.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Decrease in  Standard Deviation | 6.39 | 4.83 | 4.38 | 4.15 | 3.95 | 3.15 | 2.61 | 2.51 | 1.50 | 0.20 |
| Correlation between  X1 & X2 | -0.91 | -0.50 | -0.31 | -0.20 | -0.10 | 0.10 | 0.20 | 0.30 | 0.50 | 0.91 |

