CPM-PERT

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| --- | --- |
| 1.0. CPM/PERT Introduction2.1. CPM Critical Path2.2. CPM Parallel Paths2.3. CPM Crashing2.4. CPM Leveling | 3.1. PERT Critical Path3.2. PERT Parallel Paths3.3. PERT Non-Critical Paths3.4. PERT Crashing3.5. PERT EMV |

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| --- | --- | --- |
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|  |  |  |
|  | Beta DistributionNormal DistributionPERT Risk AnalysisExcel Spreadsheets |  |
|  |  |  |

**Project: Responding to RFP**

|  |  |
| --- | --- |
|  | **Activity List** |
| **Activity** | **Description of Activities** |
| 1 | Organize RFP requirements |
| 2 | Identify internal constraints and capabilities |
| 3 | Introduction (Background, justification, and approach) |
| 4 | SOW (Statement of Work) |
| 5 | Satisfy regulation compliance |
| 6 | Prepare document |
| 7 | Obtain approvals |

|  |
| --- |
| **Activity (A), Predecessor Activities (PA), Duration Estimates (T)** |
| **A** | **Description** | **PA** | **T** |
| 1 | Organize RFP requirements | --- | 5 days |
| 2 | Identify internal constraints and capabilities | --- | 2 days |
| 3 | Introduction (Background, justification, and approach) | 1 | 2 days |
| 4 | SOW (Statement of Work) | 2 | 3 days |
| 5 | Satisfy regulation compliance | 2 | 3 days |
| 6 | Prepare document | 3,4 | 3 days |
| 7 | Obtain approvals | 3,4,5 | 1 day |

**Project Definition**

|  |  |  |
| --- | --- | --- |
| **A** | **PA** | **T** |
| 1 | -- | 5 |
| 2 | -- | 2 |
| 3 | 1 | 2 |
| 4 | 2 | 3 |
| 5 | 2 | 3 |
| 6 | 3,4 | 3 |
| 7 | 3,4,5 | 1 |

**Network Formats.**

**Activity on Arrow (AOA). Technical content of activities.**

[Cost, Labor, Resources, Risk]



|  |  |  |
| --- | --- | --- |
| **A** | **PA** | **T** |
| 1 | -- | 5 |
| 2 | -- | 2 |
| 3 | 1 | 2 |
| 4 | 2 | 3 |
| 5 | 2 | 3 |
| 6 | 3,4 | 3 |
| 7 | 3,4,5 | 1 |

**Activity on Node (AON). Relational content of activities.**

[Finish to Start, Finish to Finish, Start to Finish, Start to Start]



|  |  |  |
| --- | --- | --- |
| **A** | **PA** | **T** |
| 1 | -- | 5 |
| 2 | -- | 2 |
| 3 | 1 | 2 |
| 4 | 2 | 3 |
| 5 | 2 | 3 |
| 6 | 3,4 | 3 |
| 7 | 3,4,5 | 1 |

**Gantt Chart. Common presentation time format for Activities.**

[All times interpreted as ‘the end of a time period’]

**Gantt Chart**

 0 1 2 3 4 5 6 7 8 9 10

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |
| 1 | -- | 5 |  | 🡪 | 🡪 | 🡪 | 🡪 | 🡪 |  |  |  |  |  |
| 2 | -- | 2 |  | 🡪 | 🡪 |  |  |  |  |  |  |  |  |
| 3 | 1 | 2 |  |  |  |  |  |  | 🡪 | 🡪 |  |  |  |
| 4 | 2 | 3 |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  |  |  |
| 5 | 2 | 3 |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  |  |  |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | 🡪 | 🡪 | 🡪 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  | 🡪 |  |  |

**Project Network Terminology**

Node = Event Branch = Activity

 Duration

 A:5

Start Finish TOC=5 days

Source Destination

Node Node

 A:5

Parallel: TOC=5 days

 B:4

 A:5 B:4

Series: TOC=9 days

Table Form:

|  |  |  |
| --- | --- | --- |
| **Activity** | **Predecessor****Activity** | **Duration** |
| A | --- | 5 |
| B | A | 4 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Activity | PredecessorActivity | Duration | EarlyStart | Late Start | Slack | CriticalPath? |
| A | --- | 4 | 0 | 0 | 0 | Yes |
| B | --- | 3 | 0 | 2 | 2 |  |
| C | A | 2 | 4 | 4 | 0 | Yes |
| D | A,B | 1 | 4 | 5 | 1 |  |

 A:4 C:2

 B:3 D:1

**Comparison of Techniques**

|  |
| --- |
| **Gantt Chart**.  |
| Time Line Plot |
| ES=Early StartEF=Early FinishTOC=Time of Completion | All time are interpreted as‘the end of a time period’. |

|  |
| --- |
| **Critical Path Method**. (CPM) |
| ES=Early StartEF=Early FinishTOC=Time of CompletionLS=Late StartLF=Late FinishSlack= LF–EF=LS–ES | All times are interpreted as‘the end of a time period’ |
| Slack=Total SlackA Critical Activity is an activity with zero slack.All critical activities define the Critical Path, CP. |

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| --- |
| **Program Evaluation and Review Technique (PERT)** |
| Three-point time estimate.t1=Optimistic Timet2=Most Likely Timet3=Pessimistic Time | E[Duration]=E[TOC]=(t1+4t2+t3)/6 V[Duration]=V[TOC]=[(t3–t1)/6]2TOC=Time of CompletionTOC~N( =E(TOC), 2=V(TOC) ) | P[TOC<T]=  |

**Critical Path Method (CPM)**



|  |  |  |
| --- | --- | --- |
| **A** | **PA** | **T** |
| 1 | -- | 5 |
| 2 | -- | 2 |
| 3 | 1 | 2 |
| 4 | 2 | 3 |
| 5 | 2 | 3 |
| 6 | 3,4 | 3 |
| 7 | 3,4,5 | 1 |

**CPM Algorithm**

|  |
| --- |
| **-----Steps-----** |
| Determine ES | Determine LF |
| Step 1. For no PA, ES=0Step 2. For all activities, EF=ES+TStep 3. For one PA, ES=EF of PA.Step 4. For more than one PA, ES=Max(EF of all PA). | Step 1. For no SA, LF=TOCStep 2. For all activities, LS= LF–T.Step 3. For one SA, LF=LS of SA.Step 4. For more than one SA, LF=Min(LS of all SA). |
| Slack=LF–EF=LS–ES |

Start with activities with no predecessors.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **A** | **Determination of ES** | **PA** | **ES** | **T** | **EF=ES+T** | **A** |
| 1 | ES=0 for no Predecessor | -- | 0 | 5 | 5 | 1 |
| 2 | ES=0 for no Predecessor | -- | 0 | 2 | 2 | 2 |
| 3 | ES=EF of Predecessor | 1 | 5 | 2 | 7 | 3 |
| 4 | ES=EF of Predecessor | 2 | 2 | 3 | 5 | 4 |
| 5 | ES=EF of Predecessor | 2 | 2 | 3 | 5 | 5 |
| 6 | ES=Max(EF of Predecessors) | 3,4 | 7 | 3 | 10 | 6 |
| 7 | ES=Max(EF of Predecessors) | 3,4,5 | 7 | 1 | 8 | 7 |
|  | TOC=Max(EF). Time of Completion (TOC) |  |  |  | 10 |  |

Invert predecessors to determine successors. SA=Successor Activity.

Start with activities with no successors.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **Determination of LF** | **SA** | **LF** | **T** | **LS=LF**–**T** | **A** |  | **Slack****(LF**–**EF)** |
| 1 | -- | LF=LS of Successor | 3 | 5 | 5 | 0 | 1 |  | 0 = 5 – 5 |
| 2 | -- | LF=Min(LS of Successors) | 4,5 | 4 | 2 | 2 | 2 |  | 2 = 4 – 2 |
| 3 | 1 | LF=Min(LS of Successors) | 6,7 | 7 | 2 | 5 | 3 |  | 0 = 7 – 7 |
| 4 | 2 | LF=Min(LS of Successors) | 6,7 | 7 | 3 | 4 | 4 |  | 2 = 7 – 5 |
| 5 | 2 | LF=LS of Successor | 7 | 9 | 3 | 6 | 5 |  | 4 = 9 – 5 |
| 6 | 3,4 | LF=TOC for no Successors | -- | 10 | 3 | 7 | 6 |  | 0 = 10 – 10 |
| 7 | 3,4,5 | LF=TOC for no Successors | -- | 10 | 1 | 9 | 7 |  | 2 = 10 – 8 |

A Critical Activity has zero slack. All Critical Activities define the Critical Path (CP)

**Network, Durations, and Paths through the Network**

**AOA (Activity On Arrow) with Critical Path**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** | Slack |  |  |  |  |
| 1 | -- | 5 | 0 |  |  |  |  |
| 2 | -- | 2 | 2 |  |  |  |  |
| 3 | 1 | 2 | 0 |  |  |  |  |
| 4 | 2 | 3 | 2 |  |  |  |  |
| 5 | 2 | 3 | 4 |  |  |  |  |
| 6 | 3,4 | 3 | 0 |  |  |  |  |
| 7 | 3,4,5 | 1 | 2 |  |  |  |  |

**AON (Activity On Node) with Critical Path**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** | Slack |  |  |  |  |  |  |  |  |
| 1 | -- | 5 | 0 |  |  |  |  |  |  |  |  |
| 2 | -- | 2 | 2 |  |  |  |  |  |  |  |  |
| 3 | 1 | 2 | 0 |  |  |  |  |  |  |  |  |
| 4 | 2 | 3 | 2 |  |  |  |  |  |  |  |  |
| 5 | 2 | 3 | 4 |  |  |  |  |  |  |  |  |
| 6 | 3,4 | 3 | 0 |  |  |  |  |  |  |  |  |
| 7 | 3,4,5 | 1 | 2 |  |  |  |  |  |  |  |  |

|  |
| --- |
| **Paths through the network:** |
|  | Paths: | Time of Completion(TOC) |
| **1** | **1-3-6** | **5+2+3=10 🡨Max TOC = Critical Path** |
| 2 | 1-3-7 | 5+2+1=8 |
| 3 | 2-4-6 | 2+3+3=8 |
| 4 | 2-4-7 | 2+3+1=6 |
| 5 | 2-5-7 | 2+3+1=6 |

**Gantt Chart with Critical Path.**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |
| 1 | -- | 5 |  | 🡺 | 🡺 | 🡺 | 🡺 | 🡺 |  |  |  |  |  |
| 2 | -- | 2 |  | 🡪 | 🡪 |  |  |  |  |  |  |  |  |
| 3 | 1 | 2 |  |  |  |  |  |  | 🡺 | 🡺 |  |  |  |
| 4 | 2 | 3 |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  |  |  |
| 5 | 2 | 3 |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  |  |  |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | 🡺 | 🡺 | 🡺 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  | 🡪 |  |  |

**CPM with Parallel Paths**

|  |
| --- |
| Change duration of Activity “2” to 4 resulting in parallel paths in the critical path. |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **A** | **PA** | **T** |  |  |
| 1 | -- | 5 |  |  |
| 2 | -- | **4** |  |  |
| 3 | 1 | 2 |  |  |
| 4 | 2 | 3 |  |  |
| 5 | 2 | 3 |  |  |
| 6 | 3,4 | 3 |  |  |
| 7 | 3,4,5 | 1 |  |  |

**Gantt Chart**

 0 1 2 3 4 5 6 7 8 9 10

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |
| 1 | -- | 5 |  | 🡺 | 🡺 | 🡺 | 🡺 | 🡺 |  |  |  |  |  |
| 2 | -- | **4** |  | 🡺 | 🡺 | 🡺 | 🡺 |  |  |  |  |  |  |
| 3 | 1 | 2 |  |  |  |  |  |  | 🡺 | 🡺 |  |  |  |
| 4 | 2 | 3 |  |  |  |  |  | 🡺 | 🡺 | 🡺 |  |  |  |
| 5 | 2 | 3 |  |  |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | 🡺 | 🡺 | 🡺 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  | 🡪 |  |  |

**CPM: Critical Path Method**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** | **ES** | **EF** | **SA** | **LF** | **LS** | **Slack** | **T–CP** |
| 1 | -- | 5 | **0** | **5** | 3 | 5 | 0 | **0** | **5** |
| 2 | -- | **4** | **0** | **4** | 4,5 | 4 | 0 | **0** | **4** |
| 3 | 1 | 2 | **5** | **7** | 6,7 | 7 | 5 | **0** | **2** |
| 4 | 2 | 3 | **4** | **7** | 6,7 | 7 | 4 | **0** | **3** |
| 5 | 2 | 3 | **4** | **7** | 7 | 9 | 6 | **2** |  |
| 6 | 3,4 | 3 | **7** | **10** | --- | 10 | 7 | **0** | **3** |
| 7 | 3,4,5 | 1 | **7** | **8** | --- | 10 | 9 | **2** |  |
|  | TOC= | **10** |  |  |  | Sum= | **17** |

 **↑** **↑**

|  |
| --- |
| When the sum of the durations of the critical activities is greater than the time of completion (TOC), then there are parallel paths in the critical path. |

**CPM: Crashing**

|  |
| --- |
| “Crashing” a project is reducing the time of completion of the project. |

Consider the project.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** | **ES** | **EF** | **SA** | **T** | **LS** | **LF** | **Slk** |
| 1 | -- | 5 | 0 | 5 | 3 | 5 | 0 | 5 | 0 |
| 2 | -- | 2 | 0 | 2 | 4,5 | 2 | 2 | 4 | 2 |
| 3 | 1 | 2 | 5 | 7 | 6,7 | 2 | 5 | 7 | 0 |
| 4 | 2 | 3 | 2 | 5 | 6,7 | 3 | 4 | 7 | 2 |
| 5 | 2 | 3 | 2 | 5 | 7 | 3 | 6 | 9 | 4 |
| 6 | 3,4 | 3 | 7 | 10 | -- | 3 | 7 | 10 | 0 |
| 7 | 3,4,5 | 1 | 7 | 8 | -- | 1 | 9 | 10 | 2 |

Assume linear cost functions.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **A** | **T** | **Cost** | **Crash T** | **Crash Cost** | **Cost Rate** | **Slack** |
| 1 | 5 | 100 | 2 | 160 | 20 | 0 |
| 2 | 2 | 40 | 1 | 55 | 15 | 2 |
| 3 | 2 | 50 | 1 | 75 | 25 | 0 |
| 4 | 3 | 60 | 2 | 75 | 15 | 2 |
| 5 | 3 | 50 | 1 | 70 | 10 | 4 |
| 6 | 3 | 60 | 1 | 120 | 30 | 0 |
| 7 | 1 | 30 | 1 | 30 | 0 | 2 |
| TOC | 10 |  |  |  |  |  |
| Cost |  | 390 |  | 585 |  |  |

|  |
| --- |
| **Objective: Crash project by 2. Crash project TOC=10 to TOC=8.** |
|  | **Option 1.****Crash Activity “1” by 2.** | **Option 2.****Crash Activity “6” by 2.** | **Option 3.****Crash Activities “1” & “6” by 1.** |
| **A** | **T** | **Cost**  | **Slack** | **T** | **Cost**  | **Slack** | **T** | **Cost**  | **Slack** |
| 1 | **3** | 140 | 0 | 5 | 100 | 0 | **4** | 120 | 0 |
| 2 | 2 | 40 | 0 | 2 | 40 | 2 | 2 | 40 | 1 |
| 3 | 2 | 50 | 0 | 2 | 50 | 0 | 2 | 50 | 0 |
| 4 | 3 | 60 | 0 | 3 | 60 | 2 | 3 | 60 | 1 |
| 5 | 3 | 50 | 2 | 3 | 50 | 2 | 3 | 50 | 2 |
| 6 | 3 | 60 | 0 | **1** | 120 | 0 | **2** | 90 | 0 |
| 7 | 1 | 30 | 2 | 1 | 30 | 0 | 1 | 30 | 1 |
| TOC | 8 |  |  | 8 |  |  | 8 |  |  |
| Cost |  | 430 |  |  | 450 |  |  | 440 |  |

**CPM: Crashing with Parallel Paths**

|  |
| --- |
| Change duration of Activity “2” to 4 resulting in parallel paths in the critical path. |



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **A** | **PA** | **T** |  |  |
| 1 | -- | 5 |  |  |
| 2 | -- | 4 |  |  |
| 3 | 1 | 2 |  |  |
| 4 | 2 | 3 |  |  |
| 5 | 2 | 3 |  |  |
| 6 | 3,4 | 3 |  |  |
| 7 | 3,4,5 | 1 |  |  |

Assume linear cost functions.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **A** | **T** | **Cost** | **Crash T** | **Crash Cost** | **Cost Rate** | **Slack** |
| 1 | 5 | 100 | 2 | 160 | 20 | 0 |
| 2 | 4 | 10 | 1 | 55 | 15 | 0 |
| 3 | 2 | 50 | 1 | 75 | 25 | 0 |
| 4 | 3 | 60 | 2 | 80 | 20 | 0 |
| 5 | 3 | 50 | 1 | 70 | 10 | 2 |
| 6 | 3 | 60 | 1 | 120 | 30 | 0 |
| 7 | 1 | 30 | 1 | 60 | 0 | 2 |
| TOC | 10 |  |  |  |  |  |
| Cost |  | 360 |  | 585 |  |  |

|  |
| --- |
| Objective: Crash project by 1. Crash from TOC=10 to TOC=9.Crash Option 1: Crash activity 2 by 1.Crash Option 2: Crash activity 6 by 1. |

|  |  |  |
| --- | --- | --- |
|  | **Option 1.**  | **Option 2.** |
| **A** | **T** | **Cost**  | **Slack** | **T** | **Cost**  | **Slack** |
| 1 | 5 | 100 | 0 | 5 | 100 | 0 |
| 2 | **3** | 25 | 1 | 4 | 10 | 0 |
| 3 | 2 | 50 | 0 | 2 | 50 | 0 |
| 4 | 3 | 60 | 1 | 3 | 60 | 0 |
| 5 | 3 | 50 | 3 | 3 | 50 | 1 |
| 6 | 3 | 60 | 0 | **2** | 90 | 0 |
| 7 | 1 | 30 | 2 | 1 | 30 | 1 |
| TOC | 10 |  |  | 9 |  |  |
| Cost |  | 375 |  |  | 390 |  |

**Rules for Crashing a Network**

|  |
| --- |
| 1. Only crash critical activities.2. Only crash one critical activity at a time.3. Only crash a critical activity by one time period at a time.4. When a critical activity is crashed, then critical activities in the network can become non-critical and non-critical activities in the network can become critical.5. When a critical activity is crashed the network may not be crashed due to parallel paths.6. Monitor crash results hierarchically.  |

Consider the example.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Activity |  |  | Activity |  | Activity |  |
| A |  |  | B |  | C |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  | Activity |  |
|  |  |  |  |  | D |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A |  |  | B |  | C |  | Activity | A | B | C | D |  |  |
| 2 |  |  | 2 |  | 2 |  | Duration | 2 | 2 | 2 | 4 | TOC |  |
|  |  |  |  |  |  |  | Slack | 0 | 0 | 0 | 0 | 6 | Original |
|  |  |  |  |  | D |  | Slack | 0 | 0 | 0 | 0 | 5 | Crash activity A by 1 |
|  |  |  |  |  | 4 |  | Slack | 0 | 1 | 1 | 0 | 5 | Crash activity B by 1 |
|  |  |  |  |  |  |  | Slack | 0 | 0 | 0 | 0 | 4 | Crash activity D by 1 |
|  |  |  |  |  |  |  | Slack | 0 | 0 | 0 | 1 | 4 | Crash activity D by 1 |

Now Consider RFP Example with parallel paths in critical path.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Activity | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |  |
| Duration | 5 | 4 | 2 | 3 | 3 | 3 | 1 | TOC |  |
| Slack | 0 | 0 | 0 | 0 | 2 | 0 | 2 | 10 | Original |
|  |  |  |  |  |  |  |  |  |  |
| Activity | 1 | 2 | 3 | 4 | 5 | 6 | 7 | TOC |  |
| Duration | 5 | 4 | 2 | 3 | 3 | 2 | 1 |  | Crash Activity 6 by 1 |
| Slack | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 9 | Activities 5 and 7 slack reduced with TOC=9 |
| Duration | 4 | 4 | 2 | 3 | 3 | 2 | 1 |  | Crash Activity 1 by 1 |
| Slack | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 9 | Activity 3 becomes non-critical with TOC=9 |
| Duration | 4 | 4 | 2 | 3 | 3 | 1 | 1 |  | Crash Activity 6 by 1 |
| Slack | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 8 | Activities 5 and 7 become critical with TOC=8 |
| Duration | 4 | 4 | 2 | 2 | 3 | 1 | 1 |  | Crash activity 4 by 1 |
| Slack | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 8 | Activities 4 and 6 become non-critical with TOC=8 |

**CPM: Resource Leveling**

|  |
| --- |
| The reallocation of slack in activities to manage fluctuations in resource requirements. |

Initially, we consider the objective of the schedule to minimize time of completion (TOC). Resource leveling introduces additional objectives or constraints on variables such as labor, inventory, equipment, and cash availability.

Consider the following Gantt chart of the ‘Early Start Schedule’ where the number of parallel activities is being examined.

1. Early Start Schedule.

|  |  |  |
| --- | --- | --- |
|  |  | **Gantt Chart** |
| **Project-1** |  |  0 1 2 3 4 5 6 7 8 9 10 |
| **A** | **PA** | **T** | **Slack** |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | -- | 5 | 0 |  | **🡪** | **🡪** | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  |
| 2 | -- | 2 | 2 |  | **🡪** | **🡪** |  |  |  |  |  |  |  |  |  |
| 3 | 1 | 2 | 0 |  |  |  |  |  |  | **🡪** | **🡪** |  |  |  |  |
| 4 | 2 | 3 | 2 |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  |
| 5 | 2 | 3 | 4 |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  |
| 6 | 3,4 | 3 | 0 |  |  |  |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  |
| 7 | 3,4,5 | 1 | 2 |  |  |  |  |  |  |  |  | **🡪** |  |  |  |
| Number of Activities | **2** | **2** | **3** | **3** | **3** | **1** | **1** | **2** | **1** | **1** |  |

Now use slack to consider other schedules to Project-1.

1. Early Start Schedule.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Gantt Chart-1a** |  |  |  |
| **Project-1** |  0 1 2 3 4 5 6 7 8 9 10 | Slack |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |  | Original | Used | Left |
| 1 | -- | 5 |  | **🡪** | **🡪** | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  | 0 | 0 | 0 |
| 2 | -- | 2 |  | **🡪** | **🡪** |  |  |  |  |  |  |  |  |  | 2 | 0 | 2 |
| 3 | 1 | 2 |  |  |  |  |  |  | **🡪** | **🡪** |  |  |  |  | 0 | 0 | 0 |
| 4 | 2 | 3 |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  | 2 | 0 | 2 |
| 5 | 2 | 3 |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  | 4 | 0 | 4 |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  | 0 | 0 | 0 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  | **🡪** |  |  |  | 2 | 0 | 2 |
| Number of Activities | **2** | **2** | **3** | **3** | **3** | **1** | **1** | **2** | **1** | **1** |  |  |  |  |

2. Late Start Schedule.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Gantt Chart-1b** |  |  |  |
| **Project-1** |  0 1 2 3 4 5 6 7 8 9 10 | Slack |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |  | Original | Used | Left |
| 1 | -- | 5 |  | **🡪** | **🡪** | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  | 0 | 0 | 0 |
| 2 | -- | 2 |  |  |  | **🡪** | **🡪** |  |  |  |  |  |  |  | 2 | 2 | 0 |
| 3 | 1 | 2 |  |  |  |  |  |  | **🡪** | **🡪** |  |  |  |  | 0 | 0 | 0 |
| 4 | 2 | 3 |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  | 2 | 2 | 0 |
| 5 | 2 | 3 |  |  |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  |  | 4 | 4 | 0 |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  | 0 | 0 | 0 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  |  |  | **🡪** |  | 2 | 2 | 0 |
| Number of Activities | **1** | **1** | **2** | **2** | **2** | **2** | **3** | **2** | **2** | **2** |  |  |  |  |

3. Level Activity Schedule.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Gantt Chart-1c** |  |  |  |
| **Project-1** |  0 1 2 3 4 5 6 7 8 9 10 | Slack |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |  | Original | Used | Left |
| 1 | -- | 5 |  | **🡪** | **🡪** | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  | 0 | 0 | 0 |
| 2 | -- | 2 |  | **🡪** | **🡪** |  |  |  |  |  |  |  |  |  | 2 | 0 | 2 |
| 3 | 1 | 2 |  |  |  |  |  |  | **🡪** | **🡪** |  |  |  |  | 0 | 0 | 0 |
| 4 | 2 | 3 |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  | 2 | 0 | 2 |
| 5 | 2 | 3 |  |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  | 4 | 3 | 1 |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  | 0 | 0 | 0 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  |  | **🡪** |  |  | 2 | 1 | 1 |
| Number of Activities | **2** | **2** | **2** | **2** | **2** | **2** | **2** | **2** | **2** | **1** |  |  |  |  |

**PERT: Program Evaluation and Review Technique**

|  |  |  |  |
| --- | --- | --- | --- |
| **A** | **PA** | **T** |  |
| 1 | -- | 5 |  |
| 2 | -- | 2 |  |
| 3 | 1 | 2 |  |
| 4 | 2 | 3 |  |
| 5 | 2 | 3 |  |
| 6 | 3,4 | 3 |  |
| 7 | 3,4,5 | 1 |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **A** | **t1=Optimistic** | **t2=Most Likely** | **t3=Pessimistic** | **E[Duration]****=(t1+4t2+t3)/6** | **V[Duration]****=[(t3–t1)/6]2** |
| 1 | 4.1 | 5 | 7.1 | 5.2 | 0.25 |
| 2 | 1.4 | 2 | 3.2 | 2.1 | 0.09 |
| 3 | 0.8 | 2 | 6.8 | 2.6 | 1.00 |
| 4 | 2.1 | 3 | 4.5 | 3.1 | 0.16 |
| 5 | 0.6 | 3 | 4.2 | 2.8 | 0.36 |
| 6 | 1.2 | 3 | 7.2 | 3.4 | 1.00 |
| 7 | 1 | 1 | 1 | 1 | 0 |

For example, for activity ‘1’: E[D] = ( 4.1 + 4(5) + 7.1 )/6 = 5.2

 V[D] = [ ( 7.1 – 4.1 )/6 ]2  = 0.25

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **A** | **E[D]** | **V[D]** | **Slack** | **Critical Path** | **E[D]** | **V[D]** |
| 1 | 5.2 | 0.25 | 0 | Yes | 5.2 | 0.25 |
| 2 | 2.1 | 0.09 | 2.6 |  |  |  |
| 3 | 2.6 | 1.00 | 0 | Yes | 2.6 | 1.00 |
| 4 | 3.1 | 0.16 | 2.6 |  |  |  |
| 5 | 2.8 | 0.36 | 5.3 |  |  |  |
| 6 | 3.4 | 1.00 | 0 | Yes | 3.4 | 1.00 |
| 7 | 1 | 0 | 2.4 |  |  |  |
|  |  |  |  | Sum | 11.2 | 2.25 |
|  |  |  |  |  | E[TOC] | V[TOC] |

Assume TOC follows a normal distribution with mean 11.2 and variance 2.25 which implies standard deviation is 1.5=sqrt(2.25). Then, for a fixed value of T,

P[TOC < T]=P[Z < ( T – 11.2 )/1.5 ]= . Let Z be the standard normal random variate.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **T** | **Z** | **P[TOC<T]= ** |  | In Excel, use function, NORMSDIST(Z). |
| 10 | –0.8 | 0.212 |  | 0.2119=NORMSDIST(–0.8) |
| 11 | –0.133 | 0.447 |  | 0.4470=NORMSDIST(–0.133) |
| 12 | +0.533 | 0.703 |  | 0.7031=NORMSDIST(+0.533) |
| 13 | +1.2 | 0.885 |  | 0.8849=NORMSDIST(+1.2) |
| 14 | +1.867 | 0.969 |  | 0.9690=NORMSDIST(+1.867) |

**Standard Normal Probability Distribution**

Let the random variable, X, follow a ‘Normal Probability Distribution’

 with a population mean,  , and a population variance, 2 , expressed as X~N(  , 

Other terminology used to for the ‘Normal Probability Distribution’ includes:

 Normal Distribution, Normal Density Function, Bell Curve, or Gaussian Distribution.

Now consider the transformation, Z = ( X -  ) /  , where E[Z]=0 and V[Z]=1.

The random variable, Z, follows a ‘Standard Normal Probability Distribution’

 with a population mean, =0, and a population variance, 2=1, expressed as Z ~ N( 0 , 1 ).

Consider the graphs:

|  |  |  |  |
| --- | --- | --- | --- |
|  |   |   |  |
|  |  |  |  |
|  X  X X X~N(  ,   |
|  |  |  |  |
|  |  |  |  |
|  Z  Z ZZ~N(  , =1Z~N(  ,  |

|  |  |
| --- | --- |
| Where  = Probability X = Normal Random Variable X = X1- = Normal Variate Z = ( X -  ) /  Z = Standard Normal Random Variable Z = Z1- = Standard Normal Variate  = 0 = Population Mean 2 = 1 = Population Variance = 1 = Population Standard Deviation  | Specifically,P[ Z < Z ] = P[ Z < Z1- ] = 1-P[ Z > Z1- ] = P[ Z <  ] =  , thus, =Z0.5P[ Z = k ] = for any constant, k.P[ Z ≤ k ] ≡ P[ Z < k ] in probability measureP[ Z < Z < Z1- ] = 1-2Empirical Rule: P[  < Z < ] = 0.6826 P[  < Z < ] = 0.9544 P[  < Z < ] = 0.9972 |
| Determine values using Excel functions. Consider P[ Z < Z] = The Excel function: ‘ =NORMSDIST ’ stands for the Standard Normal Distribution.So, for P[ Z < Z] =  , the Excel function, =NORMSDIST( Z) , returns The Excel function: ‘ =NORMSINV ’ stands for the Standard Normal Inverse.So, for P[ Z < Z] =  , the Excel function, =NORMSINV(  ) , returns Z |

**Probabilities with PERT**

|  |
| --- |
| Consider the values from the PERT analysis of the project:Let TOC be a normally distributed random variable, TOC~N( =11.2 , 2=2.25 ) = E[TOC]=11.2 , Expected time of completion or Mean time of completion (TOC)2 = V[TOC]=2.25 , Variance of the time of completion (TOC) = SD[TOC]=1.5 , Standard Deviation of the time of completion (TOC) which is sqrt(Variance) |
| Now for a constant, K, as a random variate of TOC, we haveP[TOC≤K]=P[Z≤ (K–E[TOC])/SD[TOC]]=P[Z≤ (K–So, let P[Z≤ (K–P[Z≤ Z where Z(K–Thus, we may obtain  = NORMSDIST( (K–  and K = + NORMSINV( ) \*  |

**Definitions.**

|  |
| --- |
| Since the expected time of completion is 11.2 days, probabilities will express the day of completion to be [11≤TOC≤12] or completed on the 12th day. Thus, the probability of completion at least one day early would be P[TOC≤11] and at least one day late P[TOC≥12]. However, we will interpret the statement “completed exactly one day or more ahead of schedule” to mean P[TOC≤10.2] which is P[TOC≤ (11.2–1)]. Similarly, the statement “completed exactly one day or more behind schedule” would be P[TOC≥12.2]. Finally, it should be noted that mathematically for continuous distributions, the expressionsP[TOC≤11.2] is equivalent to P[TOC<11.2] & P[TOC≥12.2] is equivalent to P[TOC>12.2] . |

**Probabilities with PERT.**

|  |
| --- |
| 1. What is the probability the project will be at least one day early?P[TOC<11]=P[Z<(11–11.2)/1.5]=P[Z<–0.133]=0.44702. What is the probability the project will be exactly one day or more behind schedule?P[TOC>12.2]=P[Z>(12.2 –11.2)/1.5]=P[Z>+0.667]=1–0.7475=0.25253. What is the probability that the project will be completed within one day of its expected time of completion? Since ‘day of completion’ is [11<TOC<12], within one day would be one day ahead and behind expressed as [11–1<TOC<12+1]. Thus,P[10<TOC<13]=P[(10–11.2)/1.5<Z<(13–11.2)/1.5]= P[–0.8<Z<+1.2] =P[Z<+1.2] – P[Z<–0.8]=0.8849–0.2119=0.6731 |

**Random Variates with PERT**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| For a constant, K, consider the relations, P[TOC≤K]=P[Z<(K–= and P[Z<Z]=Equating the random variates and solving yields, **K=  + Z \* **Common Standard Normal Variates, Z , are

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0.9 | 0.95 | 0.975 | 0.99 | 0.8413 | 0.9332 | 0.9772 | 0.9938 | 0.9987 |
| Z | 1.282 | 1.645 | 1.960 | 2.326 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |

Consider the problems.1. What time of completion yields a 90% probability of meeting and a 10% probability of exceeding?TOC0.9= + Z \* ≈≈ days2. What is a 90% confidence interval of the time of completion?(TOC0.05 , TOC0.95 ) ≈ (–≈3. What confidence results in an interval of mean plus or minus two standard deviations?Level of confidence = (2–1) ≈ (2\*0.9772–1) ≈ 0.9544. A 95.44% confidence interval. |

**PERT with Parallel Paths**

|  |
| --- |
| Change Duration Estimates of Activity 5. |



|  |  |
| --- | --- |
| **A** | **PA** |
| 1 | -- |
| 2 | -- |
| 3 | 1 |
| 4 | 2 |
| 5 | 2 |
| 6 | 3,4 |
| 7 | 3,4,5 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **A** | **t1=Optimistic** | **t2=Most Likely** | **t3=Pessimistic** | **E[Duration]****=(t1+4t2+t3)/6** | **V[Duration]****=[(t3–t1)/6]2** |  |
| 1 | 4.1 | 5 | 7.1 | 5.2 | 0.25 |  |
| 2 | 1.4 | 2 | 3.2 | 2.1 | 0.09 |  |
| 3 | 0.8 | 2 | 6.8 | 2.6 | 1.00 |  |
| 4 | 2.1 | 3 | 4.5 | 3.1 | 0.16 |  |
| **5** | **5.2** | **7** | **15.4** | **8.1** | **2.89** | 🡨Change |
| 6 | 1.2 | 3 | 7.2 | 3.4 | 1.00 |  |
| 7 | 1 | 1 | 1 | 1 | 0 |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **┌** | **CPM** | **┐** | **Critical Path** |  |
|  | **↓** |  | **↓** | **Path 1-3-6** | **Path 2-5-7** |  |
| **A** | **E[D]** | **V[D]** | **Slack** | **E[D]** | **V[D]** | **E[D]** | **V[D]** |  |
| 1 | 5.2 | 0.25 | 0 | 5.2 | 0.25 |  |  |  |
| 2 | 2.1 | 0.09 | 0 |  |  | 2.1 | 0.09 |  |
| 3 | 2.6 | 1.00 | 0 | 2.6 | 1.00 |  |  |  |
| 4 | 3.1 | 0.16 | 2.6 |  |  |  |  | 🡨Non-Critical |
| 5 | 8.1 | 2.89 | 0 |  |  | 8.1 | 2.89 |  |
| 6 | 3.4 | 1.00 | 0 | 3.4 | 1.00 |  |  |  |
| 7 | 1 | 0 | 0 |  |  | 1 | 0 |  |
|  |  |  | Sum | 11.2 | 2.25 | 11.2 | 2.98 |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **T=Day:** | **9** | **10** | **11** | **11.2** | **12** | **13** | **14** |
| Z136 | -1.467 | -0.800 | -0.133 | 0 | 0.533 | 1.200 | 1.867 |
| P[TOC136 < T]= | 0.071 | 0.212 | 0.447 | 0.5 | 0.703 | 0.885 | 0.969 |
| Z257 | -1.274 | -0.695 | -0.116 | 0 | 0.463 | 1.043 | 1.622 |
| P[TOC257 < T]= | 0.101 | 0.243 | 0.454 | 0.5 | 0.678 | 0.851 | 0.948 |
| P[TOC < T]= | 0.007 | 0.052 | 0.203 | 0.25 | 0.477 | 0.753 | 0.918 |

**Non-critical Activities in PERT with Parallel Paths**

**Consider project time of completion (TOC) probabilities in more detail.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1. All activities are stochastic except activity “7” since it has a variance of zero.2. All activities are critical except activity “4” since it has a positive slack.3. Critical Path consists of activities (1,3,6) & (2,5,7) which represent two paths.4. But the project consists of five paths, (1,3,6), (1,3,7), (2,4,6), (2,4,7), (2,5,7).5. The probability of the project time of completion depends on all the paths.6. For ease of calculation and to simplify illustration, assume all paths are parallel.7. Consider the probability results:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Path | Mean | Variance | Time= | 9 | 10 | 11 | 12 | 13 | 14 |
| 136 | 11.2 | 2.25 | Probability= | 0.071 | 0.212 | 0.447 | 0.703 | 0.885 | 0.969 |
| 137 | 8.8 | 1.25 | Probability= | 0.571 | 0.858 | 0.975 | 0.998 | 1.000 | 1.000 |
| 246 | 8.6 | 1.25 | Probability= | 0.640 | 0.895 | 0.984 | 0.999 | 1.000 | 1.000 |
| 247 | 6.2 | 0.25 | Probability= | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 257 | 11.2 | 2.98 | Probability= | 0.101 | 0.243 | 0.454 | 0.678 | 0.851 | 0.948 |
|  |  | Project | Probability= | 0.003 | 0.040 | 0.195 | 0.475 | 0.753 | 0.918 |

Observations:1. As variances increase for any activity or as the mean duration increases for any non-critical activity, the project probability will tend to decrease.2. As the number of paths increase, the project probability will tend to decrease.3. Critical activities and non-critical activities are both important.. |

Expected Monetary Value (EMV)

**PERT with EMV.**

Consider a company is based on the project.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **t1** | **t2** | **t3** | **E[Duration]****=(t1+4t2+t3)/6** | **V[Duration]****=[(t3–t1)/6]2** |  | **From PERT Analysis** |
| 1 | 4.1 | 5 | 7.1 | 5.2 | 0.25 |  | T | Z | P[TOC≤T] |
| 2 | 1.4 | 2 | 3.2 | 2.1 | 0.09 |  | 9.000 | -1.467 | 0.071 |
| 3 | 0.8 | 2 | 6.8 | 2.6 | 1.00 |  | 10.000 | -0.800 | 0.212 |
| 4 | 2.1 | 3 | 4.5 | 3.1 | 0.16 |  | 11.000 | -0.133 | 0.447 |
| 5 | 0.6 | 3 | 4.2 | 2.8 | 0.36 |  | 12.000 | 0.533 | 0.703 |
| 6 | 1.2 | 3 | 7.2 | 3.4 | 1.00 |  | 13.000 | 1.200 | 0.885 |
| 7 | 1 | 1 | 1 | 1 | 0 |  | E[TOC]=11.2 & V[TOC]=2.25 |

In responding to an RFP, a company is offered two options to include in its project proposal. The time of completion of the project is expected to be on the 12th day (i.e., completed between 11 and 12 days as measured on a continuous scale, specifically, P[11≤TOC≤12], since completing the project in 11.2 days would be on the 12th day ). The two options for bonus and penalty schedules are reported:

|  |  |  |
| --- | --- | --- |
| Option 1. | Option 2. |  |
| If completed | Payout | If completed | Payout | Probabilities |
|  |  |  |  |  |
| At least 2 days early | $400 | At least 2 days early | $350 | P[TOC≤10]=0.212 |
| 1 day early | $300 | 1 day early | $250 | P[10≤TOC≤11]=0.447-0.212=0.235 |
| On time, Day 12 | $200 | On time, Day 12 | $200 | P[11≤TOC≤12]=0.703-0.447=0.256 |
| 1 day late | $0 | 1 day late | $100 | P[12≤TOC≤13]=0.885-0.703=0.182 |
| At least 2 days late | $0 | At least 2 days late | $50 | P[TOC≥13]=1-0.885=0.115 |

The company conducted a PERT analysis on Excel and reported the following results.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **From PERT Analysis** |  | Option 1. |  | Option 2. |
| Min | Max | Prob |  | $ | EMV |  | $ | EMV |
|  |  |  |  |  |  |  |  |  |
|  | 10 | 0.212 |  | $400 | $84.80 |  | $350 | $74.20 |
| 10 | 11 | 0.235 |  | $300 | $70.50 |  | $250 | $58.75 |
| 11 | 12 | 0.256 |  | $200 | $51.20 |  | $200 | $51.20 |
| 12 | 13 | 0.182 |  | $0 | $0 |  | $100 | $18.20 |
| 13 |  | 0.115 |  | $0 | $0 |  | $50 | $5.75 |
|  | Sum= | 1.000 |  | Total= | $206.50 |  | Total= | $208.10 |

Discussion:

Note 1. Consider variables that would create a basis for strategic analysis.

Note 2. Identify characteristics that would support selection of variables to modify.

Note 2. Extend analysis to include ‘risk’ and ‘utility’ within the decision process.