**PERT – Program Evaluation Review Technique**

Michael D. Harper, Ph.D.

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|  | **CPM** | **PERT** |
| 1 | CPM – Algorithm | PERT – Algorithm |
| 2 | CPM – Parallel Paths | PERT – Probabilities |
| 3 | CPM – Crashing  Rules for Crashing a Network | PERT – Parallel Paths |
| 4 | CPM – Fast Tracking | PERT – Non-critical Paths |
| 5 | CPM – Resource Leveling | PERT – EMV  EMV, Crashing, Non-critical Paths |

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|  | **PERT Algorithm** |  |

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|  | **PERT Probabilities** |  |

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|  | **PERT Parallel Paths** |  |

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|  | **PERT Non-critical Paths** |  |

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|  | **PERT EMV (Expected Monetary Value)** |  |

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|  | **PERT Algorithm** |  |

**PERT: Program Evaluation and Review Technique**

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| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |
| 1 | -- | 5 |  | 1 |  | 3 |  | 6 |  |  |  |
| 2 | -- | 2 |  |  |  |  |  |  |  |  |  |
| 3 | 1 | 2 |  | 2 |  | 4 |  |  |  |  |  |
| 4 | 2 | 3 |  |  |  |  |  |  |  |  |  |
| 5 | 2 | 3 |  |  |  | 5 |  | 7 |  |  |  |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  |  |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  |  |

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| **A** | **t1=Optimistic** | **t2=Most Likely** | **t3=Pessimistic** | **E[Duration]**  **=(t1+4t2+t3)/6** | **V[Duration]**  **=[(t3–t1)/6]2** |
| 1 | 4.1 | 5 | 7.1 | 5.2 | 0.25 |
| 2 | 1.4 | 2 | 3.2 | 2.1 | 0.09 |
| 3 | 0.8 | 2 | 6.8 | 2.6 | 1.00 |
| 4 | 2.1 | 3 | 4.5 | 3.1 | 0.16 |
| 5 | 0.6 | 3 | 4.2 | 2.8 | 0.36 |
| 6 | 1.2 | 3 | 7.2 | 3.4 | 1.00 |
| 7 | 1 | 1 | 1 | 1 | 0 |

For example, for activity ‘1’: E[D] = ( 4.1 + 4(5) + 7.1 )/6 = 5.2

V[D] = [ ( 7.1 – 4.1 )/6 ]2  = 0.25

Perform CPM Algorithm to obtain Slack.

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| **A** | **E[D]** | **V[D]** | **Slack** | **Critical Path** | **E[D]** | **V[D]** |
| 1 | 5.2 | 0.25 | 0 | Yes | 5.2 | 0.25 |
| 2 | 2.1 | 0.09 | 2.6 |  |  |  |
| 3 | 2.6 | 1.00 | 0 | Yes | 2.6 | 1.00 |
| 4 | 3.1 | 0.16 | 2.6 |  |  |  |
| 5 | 2.8 | 0.36 | 5.3 |  |  |  |
| 6 | 3.4 | 1.00 | 0 | Yes | 3.4 | 1.00 |
| 7 | 1 | 0 | 2.4 |  |  |  |
|  |  |  |  | Sum | 11.2 | 2.25 |
|  |  |  |  |  | E[TOC] | V[TOC] |

Assume TOC follows a normal distribution with mean 11.2 and variance 2.25 which implies standard deviation is 1.5=sqrt(2.25). Then, for a fixed value of T,

P[TOC < T]= P[Z < (T–11.2)/1.5 ]= P[Z < Z]=  where Z(T–11.2)/1.5 ]

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| --- | --- | --- | --- | --- |
| **T** | **Z**  **=(T–E[TOC])/sqrt(V[TOC])** | **P[TOC<T]**  **= ** |  | **In Excel, use function,**  **NORMSDIST(Z).** |
| 10 | –1.2/1.5 = –0.8 | 0.21186 |  | 0.21186≈NORMSDIST(–1.2/1.5) |
| 11 | –0.2/1.5 ≈ –0.13333 | 0.44696 |  | 0.44696≈NORMSDIST(–0.2/1.5) |
| 12 | +0.8/1.5 ≈ 0.53333 | 0.70310 |  | 0.70310≈NORMSDIST(+0.8/1.5) |
| 13 | +1.8/1.5 = 1.2 | 0.88493 |  | 0.88493≈NORMSDIST(+1.8/1.5) |
| 14 | +2.8/1.5 ≈ 1.86667 | 0.96903 |  | 0.96903≈NORMSDIST(+2.8/1.5) |

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|  | **PERT Probabilities** |  |

**Normal Probability Distribution**

Let the random variable, X, follow a ‘Normal Probability Distribution.’

Other terminology used to for the ‘Normal Probability Distribution’ includes:

Normal Distribution, Normal Density Function, Bell Curve, or Gaussian Distribution.

Consider the Normal graph:

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|  |  |  | |  |
| X  X X  X~N(  ,  | | | | |
| Where   = Probability  X = Random Variable  X = X1- = Random Variate   = Population Mean = E[X]  2 = Population Variance = V[X]   = Population Standard Deviation | | | Specifically,  P[ X < X ] =   P[ X < X1- ] = 1-  P[ X > X1- ] =   P[ X <  ] =  , thus, =X0.5  P[ X =  ] = and for any constant.  P[ X < X < X1- ] = 1-2  Empirical Rule:  P[ - < X < +] ≈ 0.6826  P[ -2 < X < +2] ≈ 0.9544  P[ -3 < X < +3] ≈ 0.9972 | |

**Standard Normal Probability Distribution**

Now consider the transformation, Z = ( X -  ) /  , where E[Z]=0 and V[Z]=1.

Thus, Z ~ N( 0 , 1 ), called the Standard Normal Distribution. Consider the graphs:

|  |  |  |  |  |
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|  |  | |  |  |
|  |  | |  |  |
| X  X X    X~N(  ,  | | | | |
|  |  | |  |  |
|  |  | |  |  |
| Z  Z Z  Z~N(  , =1  Z~N(  ,  | | | | |
| Where   = Probability  Z = Standard Normal Random Variable  Z = Z1- = Standard Normal Variate   = 0 = Population Mean  2 = 1 = Population Variance   = 1 = Population Standard Deviation | | Specifically,  P[ Z < Z ] =   P[ Z < Z1- ] = 1-  P[ Z > Z1- ] =   P[ Z <  ] =  , thus, =X0.5  P[ X =  ] = and for any constant.  P[ Z < Z < Z1- ] = 1-2 | | |
| Empirical Rule:  P[  < Z < ] ≈ 0.6826 = NORMSDIST(1) - NORMSDIST(-1)  P[  < Z < ] ≈ 0.9544 = NORMSDIST(2) - NORMSDIST(-2)  P[  < Z < ] ≈ 0.9972 = NORMSDIST(3) - NORMSDIST(-3) | | | | |

**Probabilities with PERT**

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| Consider the values from the PERT analysis of the project:  Let TOC be a normally distributed random variable   = E[TOC]=11.2 , Expected time of completion or Mean time of completion  2 = V[TOC]=2.25 , Variance of the time of completion   = SD[TOC]=1.5 , Standard Deviation of the time of completion = sqrt(Variance)  Also, Z=(TOC–E[TOC])/SD[TOC] = (TOC –  Thus, for a constant, K, P[TOC<=K]=P[Z<=(K–E[TOC])/SD[TOC]]=P[Z<(K– |

**Definitions.**

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| Since the expected time of completion is 11.2 days, probabilities will express the day of completion to be [11<=TOC<=12] or completed on the 12th day. Thus, the probability of completion at least one day early would be P[TOC<=11] and at least one day late P[TOC>=12]. However, we will interpret the statement “completed exactly one day or more ahead of schedule” to mean P[TOC<=10.2] which is P[TOC<=(11.2–1)]. Similarly, the statement “completed exactly one day or more behind schedule” would be P[TOC>=12.2]. Finally, it should be noted that mathematically for continuous distributions, the two expressions P[TOC<=11.2] and P[TOC<11.2] are equivalent. |

**Consider the questions.**

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| 1. What is the probability the project will be at least one day early?  P[TOC<11]=P[Z<(11–11.2)/1.5]=P[Z<–0.133]=0.4470 “= NORMSDIST(-0.2/1.5)”  {Note: Probabilities such as 0.4470 can be obtained with the Excel function “=NORMSDIST(-0.2/1.5)” which stands for the “Standard Normal Distribution”.}  2. What is the probability the project will be exactly one day or more behind schedule?  P[TOC>12.2]=P[Z>(12.2 –11.2)/1.5]=P[Z>+0.667]=1–0.7475=0.2525 “=NORMSDIST(1.0/1.5)”  3. What is the probability that the project will be completed within one day of its expected time of completion? Since ‘day of completion’ is [11<TOC<12], within one day would be one day ahead and behind expressed as [11–1<TOC<12+1]. Thus,  P[10<TOC<13]=P[(10–11.2)/1.5<Z<(13–11.2)/1.5]= P[–0.8<Z<+1.2]  =P[Z<+1.2] – P[Z<–0.8]=0.8849–0.2119=0.6731 “=NORMSDIST(1.2)-NORMSDIST(-0.8)” |

**Random Variates with PERT**

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| For a constant, K, consider the relations, P[TOC<=K]=P[Z<(K–= and P[Z<Z]=  Equating the random variates and solving yields, **K=  + Z \* **  Common Standard Normal Variates, Z , are   |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  | 0.9 | 0.95 | 0.975 | 0.99 | 0.8413 | 0.9332 | 0.9772 | 0.9938 | 0.9987 | | Z | 1.282 | 1.645 | 1.960 | 2.326 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |   For Example: Z= NORMSINV(0.9) ≈ 1.282  Consider the problems.  1. What time of completion yields a 90% probability of meeting and a 10% probability of exceeding? TOC0.9= + Z \* ≈≈ days  2. What is a 90% confidence interval of the time of completion?  (TOC0.05 , TOC0.95 ) ≈ (–≈   3. What confidence results in an interval of mean plus or minus two standard deviations?  Level of confidence =(2–1) ≈ (2\*0.9772–1) ≈ 0.9544. A 95.44% confidence interval. |

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|  | **PERT Parallel Paths** |  |

**Change Duration Estimates of Activity 5.**

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| **A** | **PA** |  |  |  |  |  |  |  |  |  |  |
| 1 | -- |  | 1 |  | 3 |  | 6 |  |  |  |  |
| 2 | -- |  |  |  |  |  |  |  |  |  |  |
| 3 | 1 |  | 2 |  | 4 |  |  |  |  |  |  |
| 4 | 2 |  |  |  |  |  |  |  |  |  |  |
| 5 | 2 |  |  |  | 5 |  | 7 |  |  |  |  |
| 6 | 3,4 |  |  |  |  |  |  |  |  |  |  |
| 7 | 3,4,5 |  |  |  |  |  |  |  |  |  |  |

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| **Change Duration Estimates of Activity 5.** | | | | | |
| **A** | **t1=Optimistic** | **t2=Most Likely** | **t3=Pessimistic** | **E[Duration]**  **=(t1+4t2+t3)/6** | **V[Duration]**  **=[(t3–t1)/6]2** |
| 1 | 4.1 | 5 | 7.1 | 5.2 | 0.25 |
| 2 | 1.4 | 2 | 3.2 | 2.1 | 0.09 |
| 3 | 0.8 | 2 | 6.8 | 2.6 | 1.00 |
| 4 | 2.1 | 3 | 4.5 | 3.1 | 0.16 |
| **5** | **5.2** | **7** | **15.4** | **8.1** | **2.89** |
| 6 | 1.2 | 3 | 7.2 | 3.4 | 1.00 |
| 7 | 1 | 1 | 1 | 1 | 0 |

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|  |  |  |  | **Critical Path** | | | |
|  |  |  |  | **Path 1-3-6** | | **Path 2-5-7** | |
| **A** | **E[D]** | **V[D]** | **Slack** | **E[D]** | **V[D]** | **E[D]** | **V[D]** |
| 1 | 5.2 | 0.25 | 0 | 5.2 | 0.25 |  |  |
| 2 | 2.1 | 0.09 | 0 |  |  | 2.1 | 0.09 |
| 3 | 2.6 | 1.00 | 0 | 2.6 | 1.00 |  |  |
| 4 | 3.1 | 0.16 | 2.6 |  |  |  |  |
| **5** | **8.1** | **2.89** | **0** |  |  | 8.1 | 2.89 |
| 6 | 3.4 | 1.00 | 0 | 3.4 | 1.00 |  |  |
| 7 | 1 | 0 | 0 |  |  | 1 | 0 |
|  |  |  | Sum | 11.2 | 2.25 | 11.2 | 2.98 |

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| **T=Day:** | **9** | **10** | **11** | **11.2** | **12** | **13** | **14** |
| Z136 | -1.467 | -0.800 | -0.133 | 0 | 0.533 | 1.200 | 1.867 |
| P[TOC136 < T]= | 0.071 | 0.212 | 0.447 | 0.5 | 0.703 | 0.885 | 0.969 |
| Z257 | -1.274 | -0.695 | -0.116 | 0 | 0.463 | 1.043 | 1.622 |
| P[TOC257 < T]= | 0.101 | 0.243 | 0.454 | 0.5 | 0.678 | 0.851 | 0.948 |
| P[TOC < T]= | 0.007 | 0.052 | 0.203 | 0.25 | 0.477 | 0.753 | 0.918 |

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|  | **PERT Non-critical Paths** |  |

**Non-critical Activities in PERT with Parallel Paths**

**Consider in more detail how the project can be delayed beyond TOC.**

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| 1. All activities are stochastic except activity “7”.  2. All activities are critical except activity “4”.  3. Critical Path consists of activities 1,3,6, & 2,5,7.  4. TOC is based on the mean duration of the critical activities.  5. Assume probability of exceeding TOC due to path “1,3,6” is 0.5  6. Assume probability of exceeding TOC due to path “2,5,7” is 0.5  7. Since only activity “4” is non-critical, consider the paths:   |  |  |  | | --- | --- | --- | | **1-3**-6  **2-4**-6 | **1-3**-7  **2-4**-7 | 2-**5**-7  2-**4**-7 |   8. If the duration of activities “2” & “4” exceed “1” & “3”, the project exceeds TOC.  9. If the duration of activity “4” exceeds activity “5”, the project exceeds TOC.  10. Assume probability of not exceeding TOC due to “2,4” exceeding “1,3” is  P[T2+T4<(5.2+2.6)]=P[Z<( (5.2+2.6) – (2.1+3.1) )/sqrt(0.09+0.16) ) ] = 2 ≈ 1  11. Assume probability of not exceeding TOC due to “4” exceeding “5” is  P[T4<8.1]=P[Z<( (8.1–3.1)/sqrt(0.16) ) ] = 1 ≈ 1  12. Therefore, the probability of not exceeding TOC is (0.5\*0.5\*1\*2) = 0.25  **CAUTION: As variances increase for any activity or as the mean duration of non-critical activities increase, the probability will decrease.** |

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| **A** | **PA** |  |  |  |  |  |  |  | Consider Paths:  1-3-6  1-3-7  2-4-6  2-4-7  2-5-7 |
| 1 | -- |  | 1 |  | 3 |  | 6 |  |
| 2 | -- |  |  |  |  |  |  |  |
| 3 | 1 |  | 2 |  | 4 |  |  |  |
| 4 | 2 |  |  |  |  |  |  |  |
| 5 | 2 |  |  |  | 5 |  | 7 |  |
| 6 | 3,4 |  |  |  |  |  |  |  |
| 7 | 3,4,5 |  |  |  |  |  |  |  |

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| **Change Duration Estimates of Activity 5.** | | | | **Critical Path** | | | |  |
| **Path 1-3-6** | | **Path 2-5-7** | |  |
| **A** | **E[D]** | **V[D]** | **Slack** | **E[D]** | **V[D]** | **E[D]** | **V[D]** |  |
| 1 | 5.2 | 0.25 | 0 | 5.2 | 0.25 |  |  |  |
| 2 | 2.1 | 0.09 | 0 |  |  | 2.1 | 0.09 |  |
| 3 | 2.6 | 1.00 | 0 | 2.6 | 1.00 |  |  |  |
| 4 | 3.1 | 0.16 | 2.6 |  |  |  |  | 🡨Non-Critical |
| **5** | **8.1** | **2.89** | **0** |  |  | 8.1 | 2.89 |  |
| 6 | 3.4 | 1.00 | 0 | 3.4 | 1.00 |  |  |  |
| 7 | 1 | 0 | 0 |  |  | 1 | 0 |  |
|  |  |  | Sum | 11.2 | 2.25 | 11.2 | 2.98 |  |

**Note: A complete analysis would include all conditions and paths.**

**PERT Analysis of Critical and Non-critical Paths**



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| **Change Activity 5.** | | | | **Critical Path** | | | |  |  |  |  |  |  |
| **Path 1-3-6** | | **Path 2-5-7** | | **Path 1-3-7** | | **Path 2-4-6** | | **Path 2-4-7** | |
| **A** | **E[D]** | **V[D]** | **Slack** | **E[D]** | **V[D]** | **E[D]** | **V[D]** | **E[D]** | **V[D]** | **E[D]** | **V[D]** | **E[D]** | **V[D]** |
| 1 | 5.2 | 0.25 | 0 | 5.2 | 0.25 |  |  | 5.2 | 0.25 |  |  |  |  |
| 2 | 2.1 | 0.09 | 0 |  |  | 2.1 | 0.09 |  |  | 2.1 | 0.09 | 2.1 | 0.09 |
| 3 | 2.6 | 1.00 | 0 | 2.6 | 1.00 |  |  | 2.6 | 1.00 |  |  |  |  |
| 4 | 3.1 | 0.16 | 2.6 |  |  |  |  |  |  | 3.1 | 0.16 | 3.1 | 0.16 |
| **5** | **8.1** | **2.89** | **0** |  |  | 8.1 | 2.89 |  |  |  |  |  |  |
| 6 | 3.4 | 1.00 | 0 | 3.4 | 1.00 |  |  |  |  | 3.4 | 1.00 |  |  |
| 7 | 1 | 0 | 0 |  |  | 1 | 0 | 1 | 0 |  |  | 1 | 0 |
|  |  |  | Sum | 11.2 | 2.25 | 11.2 | 2.98 | 8.8 | 1.25 | 8.6 | 1.25 | 6.2 | 0.25 |

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|  |  |  |  |  |  |  |  |  |  |
|  | Paths | 1-3-6 | 2-5-7 | 1-3-7 | 2-4-6 | 2-4-7 |  |  |  |
|  | E[TOC] | 11.2 | 11.2 | 8.8 | 8.6 | 6.2 |  |  |  |
|  | V[TOC] | 2.25 | 2.98 | 1.25 | 1.25 | 0.25 |  |  |  |
|  | SD[TOC] | 1.5 | 1.72627 | 1.11803 | 1.11803 | 0.5 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  | T | P[TOC<T] | P[TOC<T] | P[TOC<T] | P[TOC<T] | P[TOC<T] |  | P[TOC<T] |  |
|  | 9 | 0.07123 | 0.10126 | 0.57099 | 0.63974 | 1 |  | 0.00263 |  |
|  | 10 | 0.21186 | 0.24348 | 0.85843 | 0.89475 | 1 |  | 0.03962 |  |
|  | 11 | 0.44696 | 0.45388 | 0.97545 | 0.98409 | 1 |  | 0.19474 |  |
|  | 11.2 | 0.5 | 0.5 | 0.98409 | 0.98998 | 1 |  | 0.24356 |  |
|  | 12 | 0.70310 | 0.67847 | 0.99790 | 0.99882 | 1 |  | 0.47547 |  |
|  | 13 | 0.88493 | 0.85146 | 0.99991 | 0.99996 | 1 |  | 0.75339 |  |
|  | 14 | 0.96903 | 0.94760 | 1.00000 | 1.00000 | 1 |  | 0.91824 |  |
|  |  |  |  |  |  |  |  |  |  |

Note: Consideration of non-critical paths become more significant when:

1. The E[TOC] of non-critical paths approach the E[TOC] of critical path

2. The V[TOC] of non-critical paths increase.

|  |  |  |
| --- | --- | --- |
|  | **PERT EMV (Expected Monetary Value)** |  |

**PERT with EMV**

Consider a company is based on the project.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **t1** | **t2** | **t3** | **E[Duration]**  **=(t1+4t2+t3)/6** | **V[Duration]**  **=[(t3–t1)/6]2** |  | **From PERT Analysis** | | |
| 1 | 4.1 | 5 | 7.1 | 5.2 | 0.25 |  | T | Z | P[TOC<T] |
| 2 | 1.4 | 2 | 3.2 | 2.1 | 0.09 |  | 9.000 | -1.46667 | 0.07123 |
| 3 | 0.8 | 2 | 6.8 | 2.6 | 1.00 |  | 10.000 | -0.80000 | 0.21186 |
| 4 | 2.1 | 3 | 4.5 | 3.1 | 0.16 |  | 11.000 | -0.13333 | 0.44696 |
| 5 | 0.6 | 3 | 4.2 | 2.8 | 0.36 |  | 12.000 | 0.53333 | 0.70310 |
| 6 | 1.2 | 3 | 7.2 | 3.4 | 1.00 |  | 13.000 | 1.20000 | 0.88493 |
| 7 | 1 | 1 | 1 | 1 | 0 |  | E[TOC]=11.2 & V[TOC]=2.25 | | |

In responding to an RFP, a company is offered two options to include in its project proposal. The time of completion of the project is expected to be on the 12th day (i.e., completed between 11 and 12 days as measured on a continuous scale, specifically, P[11<=TOC<=12], since completing the project in 11.2 days would be on the 12th day ). The two options for bonus and penalty schedules are reported:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Option 1. | |  | Option 2. | |
| If completed | Payout |  | If completed | Payout |
| At least 2 days early | $150 |  |  |  |
| 1 day early | $120 |  | At least 1 day early | $180 |
| On time, Day 12 | $100 |  | On time, Day 12 | $50 |
| 1 day late | $50 |  | At least 1 day late | $0 |
| At least 2 days late | $0 |  |  |  |

The company conducted a PERT analysis on Excel and reported the following results.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| From PERT Analysis | | | |  | Option 1. | |  | Option 2. | |  |
| Min | Max | Day | Prob |  | $ | EMV |  | $ | EMV |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  | 10 | <11 | 0.21186 |  | 150 | 31.778 |  | 180 | 38.134 |  |
| 10 | 11 | 11 | 0.23511 |  | 120 | 28.213 |  | 180 | 42.320 |  |
| 11 | 12 | 12 | 0.25613 |  | 100 | 25.613 |  | 50 | 12.807 |  |
| 12 | 13 | 13 | 0.18183 |  | 50 | 9.092 |  | 0 | 0.000 |  |
| 13 |  | >13 | 0.11507 |  | 0 | 0.000 |  | 0 | 0.000 |  |
|  |  | Sum= | 1.000 |  | Total | 94.696 |  | Total | 93.260 |  |

Note 1. Which variables would create a basis for strategic analysis.

Note 2. Discuss characteristics that would support selection of variables to modify.

Note 2. Along with ‘risk’ consider ‘utility’ within the decision process.

**PERT with EMV and Crashing**

In responding to an RFP, a company has selected option 1 to include in its project proposal. The RFP expects the time of completion to be on the 12th day (i.e., completed between 11 and 12 days as measured on a continuous scale, specifically, P[11<=TOC<=12], since completing the project in 11.2 days would be on the 12th day ).

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Option 1. | |  | From PERT Analysis | | | |  | Option 1. | |  |
|  |  |  | Min | Max | Day | Prob |  | $ | EMV |  |
| If completed | Payout |  |  |  |  |  |  |  |  |  |
| At least 2 days early | $150 |  |  | 10 | <11 | 0.21186 |  | 150 | 31.778 |  |
| 1 day early | $120 |  | 10 | 11 | 11 | 0.23511 |  | 120 | 28.213 |  |
| On time, Day 12 | $100 |  | 11 | 12 | 12 | 0.25613 |  | 100 | 25.613 |  |
| 1 day late | $50 |  | 12 | 13 | 13 | 0.18183 |  | 50 | 9.092 |  |
| At least 2 days late | $0 |  | 13 |  | >13 | 0.11507 |  | 0 | 0.000 |  |
|  |  |  |  |  | Sum= | 1.000 |  | Total | 94.696 |  |

The company is in the process of analyzing approaches to the proposal. Initially, the consideration is the offset of cost of crashing with bonuses for early completion. The estimated crashing cost to crash all the duration estimates for an activity by 10% would be between $2/day and $3/day based on the most likely duration estimates. Only one activity will be crashed for only one day.

The company conducted a PERT analysis on Excel similar to the above analysis considering only option 1 and reported the following results.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | Slack and Cost after | | | |
|  |  |  |  | Crashing Activity by 10% | | | |
| A | T1 | T2 | T3 | 0 | Activity 1 | Activity 3 | Activity 6 |
| 1 | 4.1 | 5 | 7.1 | 0 | 0 | 0 | 0 |
| 2 | 1.4 | 2 | 3.2 | 2.6 | 2.08 | 2.34 | 2.6 |
| 3 | 0.8 | 2 | 6.8 | 0 | 0 | 0 | 0 |
| 4 | 2.1 | 3 | 4.5 | 2.6 | 2.08 | 2.34 | 2.6 |
| 5 | 0.6 | 3 | 4.2 | 5.3 | 4.78 | 5.04 | 4.96 |
| 6 | 1.2 | 3 | 7.2 | 0 | 0 | 0 | 0 |
| 7 | 1 | 1 | 1 | 2.4 | 2.4 | 2.4 | 2.06 |
|  |  |  | EMV | 94.696 | 109.115 | 102.736 | 104.938 |
|  |  | $2/day | Cost |  | 10 | 4 | 6 |
|  |  |  | Total |  | 99.115 | 98.736 | 98.938 |
|  |  | $3/day | Cost |  | 15 | 6 | 9 |
|  |  |  | Total |  | 94.115 | 96.736 | 95.938 |
|  |  |  |  |  |  |  |  |

Consider: Cost, Slack, Critical Activities, Risk

Consider: Approach, Analysis, Strategy, Tactics