**CPM – Critical Path Method**

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|  | **CPM** | **PERT** |
| 1 | CPM – Algorithm | PERT – Algorithm |
| 2 | CPM – Parallel Paths | PERT – Probabilities |
| 3 | CPM – Crashing  Rules for Crashing a Network | PERT – Parallel Paths |
| 4 | CPM – Fast Tracking | PERT – Non-critical Paths |
| 5 | CPM – Resource Leveling | PERT – EMV  EMV, Crashing, Non-critical Paths |

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|  | **CPM Algorithm** |  |

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|  | **CPM Parallel Paths** |  |

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|  | **CPM Crashing** |  |

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|  | **CPM Fast Tracking** |  |

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|  | **CPM Resource Leveling** |  |

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|  | **CPM Algorithm** |  |

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| **Project-1** | | | **AOA (Activity On Arrow)** | | | | | | | | | | | | | | | | | | | |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | -- | 5 |  |  |  |  | 1 |  |  |  |  | 3 |  |  |  |  |  | 6 |  |  |  |  |
| 2 | -- | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 | 1 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | 2 | 3 |  |  |  |  | 2 |  |  |  |  | 4 |  |  |  |  |  |  |  |  |  |  |
| 5 | 2 | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  |  | 5 |  |  |  |  |  | 7 |  |  |  |  |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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**CPM Algorithm**

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| **-----Steps-----** | |
| Determine ES | Determine LF |
| Step 1. For no PA, ES=0  Step 2. For all activities, EF=ES+T  Step 3. For one PA, ES=EF of PA.  Step 4. For more than one PA,  ES=Max(EF of all PA). | Step 1. For no SA, LF=TOC  Step 2. For all activities, LS= LF–T.  Step 3. For one SA, LF=LS of SA.  Step 4. For more than one SA,  LF=Min(LS of all SA). |
| Slack=LF–EF=LS–ES | |

Start with activities with no predecessors.

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| **A** | **Determination of ES** | **PA** | **ES** | **T** | **EF=ES+T** | **A** |
| 1 | ES=0 for no Predecessor | -- | 0 | 5 | 5 | 1 |
| 2 | ES=0 for no Predecessor | -- | 0 | 2 | 2 | 2 |
| 3 | ES=EF of Predecessor | 1 | 5 | 2 | 7 | 3 |
| 4 | ES=EF of Predecessor | 2 | 2 | 3 | 5 | 4 |
| 5 | ES=EF of Predecessor | 2 | 2 | 3 | 5 | 5 |
| 6 | ES=Max(EF of Predecessors) | 3,4 | 7 | 3 | 10 | 6 |
| 7 | ES=Max(EF of Predecessors) | 3,4,5 | 7 | 1 | 8 | 7 |
|  | TOC=Max(EF). Time of Completion (TOC) |  |  |  | 10 |  |

Invert predecessors to determine successors. SA=Successor Activity.

Start with activities with no successors.

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| **A** | **PA** | **Determination of LF** | **SA** | **LF** | **T** | **LS=LF**–**T** | **A** |  | **Slack**  **(LF**–**EF)** |
| 1 | -- | LF=LS of Successor | 3 | 5 | 5 | 0 | 1 |  | 0 = 5 – 5 |
| 2 | -- | LF=Min(LS of Successors) | 4,5 | 4 | 2 | 2 | 2 |  | 2 = 4 – 2 |
| 3 | 1 | LF=Min(LS of Successors) | 6,7 | 7 | 2 | 5 | 3 |  | 0 = 7 – 7 |
| 4 | 2 | LF=Min(LS of Successors) | 6,7 | 7 | 3 | 4 | 4 |  | 2 = 7 – 5 |
| 5 | 2 | LF=LS of Successor | 7 | 9 | 3 | 6 | 5 |  | 4 = 9 – 5 |
| 6 | 3,4 | LF=TOC for no Successors | -- | 10 | 3 | 7 | 6 |  | 0 = 10 – 10 |
| 7 | 3,4,5 | LF=TOC for no Successors | -- | 10 | 1 | 9 | 7 |  | 2 = 10 – 8 |

A Critical Activity has zero slack. All Critical Activities define the Critical Path (CP)

**Critical Path and Paths through the Network**

**Gantt Chart, AOA (Activity On Arrow), AON (Activity On Node)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | |  | **Gantt Chart** | | | | | | | | | | | |
| **Project-1** | | |  | 0 1 2 3 4 5 6 7 8 9 10 | | | | | | | | | | | |
| **A** | **PA** | **T** | **Slack** |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | -- | 5 | 0 |  | **🡺** | **🡺** | **🡺** | **🡺** | **🡺** |  |  |  |  |  |  |
| 2 | -- | 2 | 2 |  | **🡪** | **🡪** |  |  |  |  |  |  |  |  |  |
| 3 | 1 | 2 | 0 |  |  |  |  |  |  | **🡺** | **🡺** |  |  |  |  |
| 4 | 2 | 3 | 2 |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  |
| 5 | 2 | 3 | 4 |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  |
| 6 | 3,4 | 3 | 0 |  |  |  |  |  |  |  |  | **🡺** | **🡺** | **🡺** |  |
| 7 | 3,4,5 | 1 | 2 |  |  |  |  |  |  |  |  | **🡪** |  |  |  |

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| **Project-1** | | |  | **AOA (Activity On Arrow)** | | | | | | | | | | | | | | | | | | | |
| **A** | **PA** | **T** | **Slack** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | -- | 5 | 0 |  |  |  |  | 1 |  |  |  |  | 3 |  |  |  |  |  | 6 |  |  |  |  |
| 2 | -- | 2 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 | 1 | 2 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | 2 | 3 | 2 |  |  |  |  | 2 |  |  |  |  | 4 |  |  |  |  |  |  |  |  |  |  |
| 5 | 2 | 3 | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 | 3,4 | 3 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7 | 3,4,5 | 1 | 2 |  |  |  |  |  |  |  |  |  | 5 |  |  |  |  |  | 7 |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| **Project-1** | | |  | **AON (Activity On Node)** | | | | | | |
| **A** | **PA** | **T** | **Slack** |  |  |  |  |  |  |  |
| 1 | -- | 5 | 0 |  | 1 |  | 3 |  | 6 |  |
| 2 | -- | 2 | 2 |  |  |  |  |  |  |  |
| 3 | 1 | 2 | 0 |  | 2 |  | 4 |  | 7 |  |
| 4 | 2 | 3 | 2 |  |  |  |  |  |  |  |
| 5 | 2 | 3 | 4 |  |  |  | 5 |  |  |  |
| 6 | 3,4 | 3 | 0 |  |  |  |  |  |  |  |
| 7 | 3,4,5 | 1 | 2 |  |  |  |  |  |  |  |

**Paths through the network:**

|  |  |  |
| --- | --- | --- |
|  | Paths: | Time of Completion(TOC) |
| **1** | **1-3-6** | **5+2+3=10 🡨Max TOC = Critical Path** |
| 2 | 1-3-7 | 5+2+1=8 |
| 3 | 2-4-6 | 2+3+3=8 |
| 4 | 2-4-7 | 2+3+1=6 |
| 5 | 2-5-7 | 2+3+1=6 |

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|  | **CPM Parallel Paths** |  |

**Change duration of Activity “2” to 4**

**resulting in parallel paths in the critical path.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Project-2** | | |  |  |  |  |
| **A** | **PA** | **T** | **ES** | **EF** | **SA** | **LS** | **LF** | **Slack** |
| 1 | -- | 5 | 0 | 5 | 3 | 0 | 5 | 0 |
| **2** | **--** | **4** | 0 | 4 | 4,5 | 0 | 4 | 0 |
| 3 | 1 | 2 | 5 | 7 | 6,7 | 5 | 7 | 0 |
| 4 | 2 | 3 | 4 | 7 | 6,7 | 4 | 7 | 0 |
| 5 | 2 | 3 | 4 | 7 | 7 | 6 | 9 | 2 |
| 6 | 3,4 | 3 | 7 | 10 | -- | 7 | 10 | 0 |
| 7 | 3,4,5 | 1 | 7 | 8 | -- | 9 | 10 | 2 |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Project-2** | | |  | **AON (Activity On Node)** | | | | | | |
| **A** | **PA** | **T** | **Slack** |  |  |  |  |  |  |  |
| 1 | -- | 5 | 0 |  | 1 |  | 3 |  | 6 |  |
| **2** | **--** | **4** | 0 |  |  |  |  |  |  |  |
| 3 | 1 | 2 | 0 |  | 2 |  | 4 |  | 7 |  |
| 4 | 2 | 3 | 0 |  |  |  |  |  |  |  |
| 5 | 2 | 3 | 2 |  |  |  | 5 |  |  |  |
| 6 | 3,4 | 3 | 0 |  |  |  |  |  |  |  |
| 7 | 3,4,5 | 1 | 2 |  |  |  |  |  |  |  |

**Paths through the network:**

|  |  |  |
| --- | --- | --- |
|  | Paths: | Time of Completion(TOC) |
| **1** | **1-3-6** | **5+2+3=10 🡨Max TOC = Critical Path** |
| 2 | 1-3-7 | 5+2+1=8 |
| **3** | **2-4-6** | **4+3+3=10 🡨Max TOC = Critical Path** |
| 4 | 2-4-7 | 4+3+1=8 |
| 5 | 2-5-7 | 4+3+1=8 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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|  | | |  | **Gantt Chart** | | | | | | | | | | | |
| **Project-2** | | |  | 0 1 2 3 4 5 6 7 8 9 10 | | | | | | | | | | | |
| **A** | **PA** | **T** | **Slack** |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | -- | 5 | 0 |  | **🡺** | **🡺** | **🡺** | **🡺** | **🡺** |  |  |  |  |  |  |
| **2** | **--** | **4** | 0 |  | **🡺** | **🡺** | **🡺** | **🡺** |  |  |  |  |  |  |  |
| 3 | 1 | 2 | 0 |  |  |  |  |  |  | **🡺** | **🡺** |  |  |  |  |
| 4 | 2 | 3 | 0 |  |  |  |  |  | **🡺** | **🡺** | **🡺** |  |  |  |  |
| 5 | 2 | 3 | 2 |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  |
| 6 | 3,4 | 3 | 0 |  |  |  |  |  |  |  |  | **🡺** | **🡺** | **🡺** |  |
| 7 | 3,4,5 | 1 | 2 |  |  |  |  |  |  |  |  | **🡪** |  |  |  |

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|  | **CPM Crashing** |  |

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| ***“Crashing” a project is reducing the time of completion of the project.*** |

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| --- | --- | --- | --- | --- | --- | --- | --- |
| **Project-1** | | |  |  |  |  |  |
| **A** | **PA** | **T** | **ES** | **EF** | **SA** | **T** | **LS** | **LF** | **Slk** |
| 1 | -- | 5 | 0 | 5 | 3 | 5 | 0 | 5 | 0 |
| 2 | -- | 2 | 0 | 2 | 4,5 | 2 | 2 | 4 | 2 |
| 3 | 1 | 2 | 5 | 7 | 6,7 | 2 | 5 | 7 | 0 |
| 4 | 2 | 3 | 2 | 5 | 6,7 | 3 | 4 | 7 | 2 |
| 5 | 2 | 3 | 2 | 5 | 7 | 3 | 6 | 9 | 4 |
| 6 | 3,4 | 3 | 7 | 10 | -- | 3 | 7 | 10 | 0 |
| 7 | 3,4,5 | 1 | 7 | 8 | -- | 1 | 9 | 10 | 2 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **A** | **T** | **Cost** | **Crash T** | **Crash Cost** | **Cost Rate** | **Slack** |
| 1 | 5 | 100 | 2 | 160 | 20 | 0 |
| 2 | 2 | 40 | 1 | 55 | 15 | 2 |
| 3 | 2 | 50 | 1 | 75 | 25 | 0 |
| 4 | 3 | 60 | 2 | 75 | 15 | 2 |
| 5 | 3 | 50 | 1 | 70 | 10 | 4 |
| 6 | 3 | 60 | 1 | 120 | 30 | 0 |
| 7 | 1 | 30 | 1 | 30 | 0 | 2 |
| TOC | 10 |  |  |  |  |  |
| Cost |  | 390 |  | 585 |  |  |

Objective: Crash project by 2. Crash from TOC=10 to TOC=8.

Crash Option 1: reduce activity 1 by 2.

Crash Option 2: reduce activity 6 by 2.

Crash Option 3: reduce activity 1 by 1 and activity 6 by 1.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Option 1.** | | | **Option 2.** | | | **Option 3.** | | |
| **A** | **T** | **Cost** | **Slack** | **T** | **Cost** | **Slack** | **T** | **Cost** | **Slack** |
| 1 | **3** | 140 | 0 | 5 | 100 | 0 | **4** | 120 | 0 |
| 2 | 2 | 40 | 0 | 2 | 40 | 2 | 2 | 40 | 1 |
| 3 | 2 | 50 | 0 | 2 | 50 | 0 | 2 | 50 | 0 |
| 4 | 3 | 60 | 0 | 3 | 60 | 2 | 3 | 60 | 1 |
| 5 | 3 | 50 | 2 | 3 | 50 | 2 | 3 | 50 | 2 |
| 6 | 3 | 60 | 0 | **1** | 120 | 0 | **2** | 90 | 0 |
| 7 | 1 | 30 | 2 | 1 | 30 | 0 | 1 | 30 | 1 |
| TOC | 8 |  |  | 8 |  |  | 8 |  |  |
| Cost |  | 430 |  |  | 450 |  |  | 440 |  |

**Rules for Crashing a Network**

|  |
| --- |
| 1. Only crash critical activities.  2. Only crash one critical activity at a time.  3. Only crash a critical activity by one time period at a time.  4. When a critical activity is crashed, then critical activities in the network can become non-critical and non-critical activities in the network can become critical.  5. When a critical activity is crashed the network may not be crashed due to parallel paths.  6. Monitor crash results hierarchically. |

Consider the example.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Activity |  |  | Activity |  | Activity |  |
| A |  |  | B |  | C |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  | Activity |  |
|  |  |  |  |  | D |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A |  |  | B |  | C |  | Activity | A | B | C | D |  |  |
| 2 |  |  | 2 |  | 2 |  | Duration | 2 | 2 | 2 | 4 | TOC |  |
|  |  |  |  |  |  |  | Slack | 0 | 0 | 0 | 0 | 6 | Original |
|  |  |  |  |  | D |  | Slack | 0 | 0 | 0 | 0 | 5 | Crash activity A by 1 |
|  |  |  |  |  | 4 |  | Slack | 0 | 1 | 1 | 0 | 5 | Crash activity B by 1 |
|  |  |  |  |  |  |  | Slack | 0 | 0 | 0 | 0 | 4 | Crash activity D by 1 |
|  |  |  |  |  |  |  | Slack | 0 | 0 | 0 | 1 | 4 | Crash activity D by 1 |

Now Consider RFP Example with parallel paths in critical path.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Activity | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |  |
| Duration | 5 | 4 | 2 | 3 | 3 | 3 | 1 | TOC |  |
| Slack | 0 | 0 | 0 | 0 | 2 | 0 | 2 | 10 | Original |
|  |  |  |  |  |  |  |  |  |  |
| Activity | 1 | 2 | 3 | 4 | 5 | 6 | 7 | TOC |  |
| Duration | 5 | 4 | 2 | 3 | 3 | 2 | 1 |  | Crash Activity 6 by 1 |
| Slack | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 9 | Activities 5 and 7 slack reduced with TOC=9 |
| Duration | 4 | 4 | 2 | 3 | 3 | 2 | 1 |  | Crash Activity 1 by 1 |
| Slack | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 9 | Activity 3 becomes non-critical with TOC=9 |
| Duration | 4 | 4 | 2 | 3 | 3 | 1 | 1 |  | Crash Activity 6 by 1 |
| Slack | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 8 | Activities 5 and 7 become critical with TOC=8 |
| Duration | 4 | 4 | 2 | 2 | 3 | 1 | 1 |  | Crash activity 4 by 1 |
| Slack | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 8 | Activities 4 and 6 become non-critical with TOC=8 |

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|  | **CPM Fast Tracking** |  |

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| Activity 1 |  | Activity 2 |
|  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Although Activity 2 is scheduled to start after Activity 1, often Activity 2 can start before the end of Activity 1.   |  |  |  |  |  | | --- | --- | --- | --- | --- | | Activity 1 | | |  |  | |  |  | |  |  |  |  |  | |  |  |  |  |  | |  |  | Activity 2 | | | |  |  |   - |
| Although not part of the definition of an activity needed for CPM/PERT, tasks can be defined within each activity.   |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Activity 1 | | | |  | Activity 2 | | | | Task 1-1 | Task 1-2 | Task 1-3 | Task 1-4 |  | Task 2-1 | Task 2-2 | Task 2-3 |   - |
|  |
| **Task relationships within an activity and between activities**  **can be used to fast track.** |
| **Strictly ordered tasks within an activity and relational tasks between activities**  Procedure: Schedule succeeding task immediately after preceding task.   |  |  |  |  |  | | --- | --- | --- | --- | --- | | Activity 1 | | | |  | | Task 1-1 | Task 1-2 | Task 1-3 | Task 1-4 |  | |  |  |  |  |  | |  | Task 2-1 |  | Task 2-2 | Task 2-3 | |  | Activity 2 | | | |   - |
| **Un-ordered tasks within an activity and relational tasks between activities**  Procedure: For tasks in activity 1, schedule shortest times first. For tasks in activity 2, schedule shortest times last.   |  |  |  |  | | --- | --- | --- | --- | | Activity 1 | | | | | Task 1-4 | Task 1-1 | Task 1-3 | Task 1-2 | |  |  |  |  | |  | Task 2-3 | Task 2-1 | Task 2-2 | |  | Activity 2 | | |   - |

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|  | **CPM Resource Leveling** |  |

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| ***The reallocation of slack in activities to manage fluctuations in resource requirements.*** |

Initially, we consider the objective of the schedule to minimize time of completion (TOC). Resource leveling introduces additional objectives or constraints on variables such as labor, inventory, equipment, and cash availability.

Consider the following Gantt chart of the ‘Early Start Schedule’ where the number of parallel activities is being examined.

1. Early Start Schedule.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | |  | **Gantt Chart** | | | | | | | | | | | |
| **Project-1** | | |  | 0 1 2 3 4 5 6 7 8 9 10 | | | | | | | | | | | |
| **A** | **PA** | **T** | **Slack** |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | -- | 5 | 0 |  | **🡪** | **🡪** | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  |
| 2 | -- | 2 | 2 |  | **🡪** | **🡪** |  |  |  |  |  |  |  |  |  |
| 3 | 1 | 2 | 0 |  |  |  |  |  |  | **🡪** | **🡪** |  |  |  |  |
| 4 | 2 | 3 | 2 |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  |
| 5 | 2 | 3 | 4 |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  |
| 6 | 3,4 | 3 | 0 |  |  |  |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  |
| 7 | 3,4,5 | 1 | 2 |  |  |  |  |  |  |  |  | **🡪** |  |  |  |
| Number of Activities | | | | | **2** | **2** | **3** | **3** | **3** | **1** | **1** | **2** | **1** | **1** |  |

Now use slack to consider other schedules to Project-1.

1. Early Start Schedule.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | **Gantt Chart-1a** | | | | | | | | | | | |  |  |  |
| **Project-1** | | | 0 1 2 3 4 5 6 7 8 9 10 | | | | | | | | | | | | Slack | | |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |  | Original | Used | Left |
| 1 | -- | 5 |  | **🡪** | **🡪** | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  | 0 | 0 | 0 |
| 2 | -- | 2 |  | **🡪** | **🡪** |  |  |  |  |  |  |  |  |  | 2 | 0 | 2 |
| 3 | 1 | 2 |  |  |  |  |  |  | **🡪** | **🡪** |  |  |  |  | 0 | 0 | 0 |
| 4 | 2 | 3 |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  | 2 | 0 | 2 |
| 5 | 2 | 3 |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  | 4 | 0 | 4 |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  | 0 | 0 | 0 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  | **🡪** |  |  |  | 2 | 0 | 2 |
| Number of Activities | | | | **2** | **2** | **3** | **3** | **3** | **1** | **1** | **2** | **1** | **1** |  |  |  |  |

2. Late Start Schedule.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | **Gantt Chart-1b** | | | | | | | | | | | |  |  |  |
| **Project-1** | | | 0 1 2 3 4 5 6 7 8 9 10 | | | | | | | | | | | | Slack | | |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |  | Original | Used | Left |
| 1 | -- | 5 |  | **🡪** | **🡪** | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  | 0 | 0 | 0 |
| 2 | -- | 2 |  |  |  | **🡪** | **🡪** |  |  |  |  |  |  |  | 2 | 2 | 0 |
| 3 | 1 | 2 |  |  |  |  |  |  | **🡪** | **🡪** |  |  |  |  | 0 | 0 | 0 |
| 4 | 2 | 3 |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  | 2 | 2 | 0 |
| 5 | 2 | 3 |  |  |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  |  | 4 | 4 | 0 |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  | 0 | 0 | 0 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  |  |  | **🡪** |  | 2 | 2 | 0 |
| Number of Activities | | | | **1** | **1** | **2** | **2** | **2** | **2** | **3** | **2** | **2** | **2** |  |  |  |  |

3. Level Activity Schedule.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | **Gantt Chart-1c** | | | | | | | | | | | |  |  |  |
| **Project-1** | | | 0 1 2 3 4 5 6 7 8 9 10 | | | | | | | | | | | | Slack | | |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |  | Original | Used | Left |
| 1 | -- | 5 |  | **🡪** | **🡪** | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  | 0 | 0 | 0 |
| 2 | -- | 2 |  | **🡪** | **🡪** |  |  |  |  |  |  |  |  |  | 2 | 0 | 2 |
| 3 | 1 | 2 |  |  |  |  |  |  | **🡪** | **🡪** |  |  |  |  | 0 | 0 | 0 |
| 4 | 2 | 3 |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  |  |  |  | 2 | 0 | 2 |
| 5 | 2 | 3 |  |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  |  |  | 4 | 3 | 1 |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | **🡪** | **🡪** | **🡪** |  | 0 | 0 | 0 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  |  | **🡪** |  |  | 2 | 1 | 1 |
| Number of Activities | | | | **2** | **2** | **2** | **2** | **2** | **2** | **2** | **2** | **2** | **1** |  |  |  |  |