CPM-PERT

|  |  |  |
| --- | --- | --- |
|  | Outline |  |
|  | 1. Introduction  1.1. Activities  1.2. Network Formats  1.3. Comparison  2. CPM  2.1. CPM – Critical Path  2.2. CPM – Parallel Paths  2.3. CPM – Crashing  2.4. CPM – Leveling  3. PERT – Introduction  3.1. PERT – Critical Path  3.2. PERT – Parallel Paths  3.3. PERT – Non-critical Paths  3.4. PERT – Crashing  3.5. PERT – EMV  3.6. PERT – Risk & Utility |  |
|  |  |  |
|  | Beta Distribution  Normal Distribution  Excel Spreadsheets |  |
|  |  |  |

**Project: Responding to RFP.**

|  |  |
| --- | --- |
|  | **Activity List** |
| **Activity** | **Description of Activities** |
| 1 | Organize RFP requirements |
| 2 | Identify internal constraints and capabilities |
| 3 | Introduction (Background, justification, and approach) |
| 4 | SOW (Statement of Work) |
| 5 | Satisfy regulation compliance |
| 6 | Prepare document |
| 7 | Obtain approvals |

|  |  |  |  |
| --- | --- | --- | --- |
| **Activity (A), Predecessor Activities (PA), Duration Estimates (T)** | | | |
| **A** | **Description** | **PA** | **T** |
| 1 | Organize RFP requirements | --- | 5 days |
| 2 | Identify internal constraints and capabilities | --- | 2 days |
| 3 | Introduction (Background, justification, and approach) | 1 | 2 days |
| 4 | SOW (Statement of Work) | 2 | 3 days |
| 5 | Satisfy regulation compliance | 2 | 3 days |
| 6 | Prepare document | 3,4 | 3 days |
| 7 | Obtain approvals | 3,4,5 | 1 day |

**Project Definition**

|  |  |  |
| --- | --- | --- |
| **A** | **PA** | **T** |
| 1 | -- | 5 |
| 2 | -- | 2 |
| 3 | 1 | 2 |
| 4 | 2 | 3 |
| 5 | 2 | 3 |
| 6 | 3,4 | 3 |
| 7 | 3,4,5 | 1 |

**Network Formats.**

**Activity on Arrow (AOA). Technical content of activities.**

[Cost, Labor, Resources, Risk]



|  |  |  |
| --- | --- | --- |
| **A** | **PA** | **T** |
| 1 | -- | 5 |
| 2 | -- | 2 |
| 3 | 1 | 2 |
| 4 | 2 | 3 |
| 5 | 2 | 3 |
| 6 | 3,4 | 3 |
| 7 | 3,4,5 | 1 |

**Activity on Node (AON). Relational content of activities.**

[Finish to Start, Finish to Finish, Start to Finish, Start to Start]



|  |  |  |
| --- | --- | --- |
| **A** | **PA** | **T** |
| 1 | -- | 5 |
| 2 | -- | 2 |
| 3 | 1 | 2 |
| 4 | 2 | 3 |
| 5 | 2 | 3 |
| 6 | 3,4 | 3 |
| 7 | 3,4,5 | 1 |

**Gantt Chart. Common presentation time format for Activities.**

[All times interpreted as ‘the end of a time period’]

**Gantt Chart**

0 1 2 3 4 5 6 7 8 9 10

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |
| 1 | -- | 5 |  | 🡪 | 🡪 | 🡪 | 🡪 | 🡪 |  |  |  |  |  |
| 2 | -- | 2 |  | 🡪 | 🡪 |  |  |  |  |  |  |  |  |
| 3 | 1 | 2 |  |  |  |  |  |  | 🡪 | 🡪 |  |  |  |
| 4 | 2 | 3 |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  |  |  |
| 5 | 2 | 3 |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  |  |  |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | 🡪 | 🡪 | 🡪 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  | 🡪 |  |  |

**Project Network Terminology**

Node = Event Branch = Activity

Duration

A:5

Start Finish TOC=5 days

Source Destination

Node Node

A:5

Parallel: TOC=5 days

B:4

A:5 B:4

Series: TOC=9 days

Table Form:

|  |  |  |
| --- | --- | --- |
| **Activity** | **Predecessor**  **Activity** | **Duration** |
| A | --- | 5 |
| B | A | 4 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Activity | Predecessor  Activity | Duration | Early  Start | Late Start | Slack | Critical  Path? |
| A | --- | 4 | 0 | 0 | 0 | Yes |
| B | --- | 3 | 0 | 2 | 2 |  |
| C | A | 2 | 4 | 4 | 0 | Yes |
| D | A,B | 1 | 4 | 5 | 1 |  |

A:4 C:2

B:3 D:1

**Comparison of Techniques.**

**Gantt Chart**.

|  |  |
| --- | --- |
| Time Line Plot | |
| ES=Early Start  EF=Early Finish  TOC=Time of Completion | All time are interpreted as  ‘the end of a time period’. |

**Critical Path Method**. (CPM)

|  |  |
| --- | --- |
| ES=Early Start  EF=Early Finish  TOC=Time of Completion  LS=Late Start  LF=Late Finish  Slack= LF–EF=LS–ES | All times are interpreted as  ‘the end of a time period’ |
| Slack=Total Slack  A Critical Activity is an activity with zero slack.  All critical activities define the Critical Path, CP. | |

**Program Evaluation and Review Technique (PERT)**

|  |  |  |
| --- | --- | --- |
| Three-point time estimate.  t1=Optimistic Time  t2=Most Likely Time  t3=Pessimistic Time | E[Duration]=E[TOC]=(t1+4t2+t3)/6  V[Duration]=V[TOC]=[(t3–t1)/6]2  TOC=Time of Completion  TOC~N( =E(TOC), 2=V(TOC) ) | P[TOC<T]=  |

**Critical Path Method (CPM)**



|  |  |  |
| --- | --- | --- |
| **A** | **PA** | **T** |
| 1 | -- | 5 |
| 2 | -- | 2 |
| 3 | 1 | 2 |
| 4 | 2 | 3 |
| 5 | 2 | 3 |
| 6 | 3,4 | 3 |
| 7 | 3,4,5 | 1 |

**CPM Algorithm**

|  |  |
| --- | --- |
| **-----Steps-----** | |
| Determine ES | Determine LF |
| Step 1. For no PA, ES=0  Step 2. For all activities, EF=ES+T  Step 3. For one PA, ES=EF of PA.  Step 4. For more than one PA,  ES=Max(EF of all PA). | Step 1. For no SA, LF=TOC  Step 2. For all activities, LS= LF–T.  Step 3. For one SA, LF=LS of SA.  Step 4. For more than one SA,  LF=Min(LS of all SA). |
| Slack=LF–EF=LS–ES | |

Start with activities with no predecessors.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **A** | **Determination of ES** | **PA** | **ES** | **T** | **EF=ES+T** | **A** |
| 1 | ES=0 for no Predecessor | -- | 0 | 5 | 5 | 1 |
| 2 | ES=0 for no Predecessor | -- | 0 | 2 | 2 | 2 |
| 3 | ES=EF of Predecessor | 1 | 5 | 2 | 7 | 3 |
| 4 | ES=EF of Predecessor | 2 | 2 | 3 | 5 | 4 |
| 5 | ES=EF of Predecessor | 2 | 2 | 3 | 5 | 5 |
| 6 | ES=Max(EF of Predecessors) | 3,4 | 7 | 3 | 10 | 6 |
| 7 | ES=Max(EF of Predecessors) | 3,4,5 | 7 | 1 | 8 | 7 |
|  | TOC=Max(EF). Time of Completion (TOC) |  |  |  | 10 |  |

Invert predecessors to determine successors. SA=Successor Activity.

Start with activities with no successors.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **Determination of LF** | **SA** | **LF** | **T** | **LS=LF**–**T** | **A** |  | **Slack**  **(LF**–**EF)** |
| 1 | -- | LF=LS of Successor | 3 | 5 | 5 | 0 | 1 |  | 0 = 5 – 5 |
| 2 | -- | LF=Min(LS of Successors) | 4,5 | 4 | 2 | 2 | 2 |  | 2 = 4 – 2 |
| 3 | 1 | LF=Min(LS of Successors) | 6,7 | 7 | 2 | 5 | 3 |  | 0 = 7 – 7 |
| 4 | 2 | LF=Min(LS of Successors) | 6,7 | 7 | 3 | 4 | 4 |  | 2 = 7 – 5 |
| 5 | 2 | LF=LS of Successor | 7 | 9 | 3 | 6 | 5 |  | 4 = 9 – 5 |
| 6 | 3,4 | LF=TOC for no Successors | -- | 10 | 3 | 7 | 6 |  | 0 = 10 – 10 |
| 7 | 3,4,5 | LF=TOC for no Successors | -- | 10 | 1 | 9 | 7 |  | 2 = 10 – 8 |

A Critical Activity has zero slack. All Critical Activities define the Critical Path (CP)

**Network, Durations, and Paths through the Network**

**AOA (Activity On Arrow) with Critical Path**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** | Slack |  |  |  |  |
| 1 | -- | 5 | 0 |  |  |  |  |
| 2 | -- | 2 | 2 |  |  |  |  |
| 3 | 1 | 2 | 0 |  |  |  |  |
| 4 | 2 | 3 | 2 |  |  |  |  |
| 5 | 2 | 3 | 4 |  |  |  |  |
| 6 | 3,4 | 3 | 0 |  |  |  |  |
| 7 | 3,4,5 | 1 | 2 |  |  |  |  |

**AON (Activity On Node) with Critical Path**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** | Slack |  |  |  |  |  |  |  |  |
| 1 | -- | 5 | 0 |  |  |  |  |  |  |  |  |
| 2 | -- | 2 | 2 |  |  |  |  |  |  |  |  |
| 3 | 1 | 2 | 0 |  |  |  |  |  |  |  |  |
| 4 | 2 | 3 | 2 |  |  |  |  |  |  |  |  |
| 5 | 2 | 3 | 4 |  |  |  |  |  |  |  |  |
| 6 | 3,4 | 3 | 0 |  |  |  |  |  |  |  |  |
| 7 | 3,4,5 | 1 | 2 |  |  |  |  |  |  |  |  |

**Paths through the network:**

|  |  |  |
| --- | --- | --- |
|  | Paths: | Time of Completion(TOC) |
| **1** | **1-3-6** | **5+2+3=10 🡨Max TOC = Critical Path** |
| 2 | 1-3-7 | 5+2+1=8 |
| 3 | 2-4-6 | 2+3+3=8 |
| 4 | 2-4-7 | 2+3+1=6 |
| 5 | 2-5-7 | 2+3+1=6 |

**Gantt Chart with Critical Path.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | | 9 | | 10 | |
| **A** | **PA** | **T** |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |
| 1 | -- | 5 |  | | 🡺 | | 🡺 | | 🡺 | | 🡺 | | 🡺 | |  | |  | |  | |  | |  | |
| 2 | -- | 2 |  | | 🡪 | | 🡪 | |  | |  | |  | |  | |  | |  | |  | |  | |
| 3 | 1 | 2 |  | |  | |  | |  | |  | |  | | 🡺 | | 🡺 | |  | |  | |  | |
| 4 | 2 | 3 |  | |  | |  | | 🡪 | | 🡪 | | 🡪 | |  | |  | |  | |  | |  | |
| 5 | 2 | 3 |  | |  | |  | | 🡪 | | 🡪 | | 🡪 | |  | |  | |  | |  | |  | |
| 6 | 3,4 | 3 |  | |  | |  | |  | |  | |  | |  | |  | | 🡺 | | 🡺 | | 🡺 | |
| 7 | 3,4,5 | 1 |  | |  | |  | |  | |  | |  | |  | |  | | 🡪 | |  | |  | |

**CPM with Parallel Paths**

|  |
| --- |
| Change duration of Activity “2” to 4  resulting in parallel paths in the critical path. |

****

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **A** | **PA** | **T** |  |  |
| 1 | -- | 5 |  |  |
| 2 | -- | **4** |  |  |
| 3 | 1 | 2 |  |  |
| 4 | 2 | 3 |  |  |
| 5 | 2 | 3 |  |  |
| 6 | 3,4 | 3 |  |  |
| 7 | 3,4,5 | 1 |  |  |

**Gantt Chart**

0 1 2 3 4 5 6 7 8 9 10

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |
| 1 | -- | 5 |  | 🡺 | 🡺 | 🡺 | 🡺 | 🡺 |  |  |  |  |  |
| 2 | -- | **4** |  | 🡺 | 🡺 | 🡺 | 🡺 |  |  |  |  |  |  |
| 3 | 1 | 2 |  |  |  |  |  |  | 🡺 | 🡺 |  |  |  |
| 4 | 2 | 3 |  |  |  |  |  | 🡺 | 🡺 | 🡺 |  |  |  |
| 5 | 2 | 3 |  |  |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | 🡺 | 🡺 | 🡺 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  | 🡪 |  |  |

**CPM: Critical Path Method**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** | **ES** | **EF** | **SA** | **LF** | **LS** | **Slack** | **T–CP** |
| 1 | -- | 5 | **0** | **5** | 3 | 5 | 0 | **0** | **5** |
| 2 | -- | **4** | **0** | **4** | 4,5 | 4 | 0 | **0** | **4** |
| 3 | 1 | 2 | **5** | **7** | 6,7 | 7 | 5 | **0** | **2** |
| 4 | 2 | 3 | **4** | **7** | 6,7 | 7 | 4 | **0** | **3** |
| 5 | 2 | 3 | **4** | **7** | 7 | 9 | 6 | **2** |  |
| 6 | 3,4 | 3 | **7** | **10** | --- | 10 | 7 | **0** | **3** |
| 7 | 3,4,5 | 1 | **7** | **8** | --- | 10 | 9 | **2** |  |
|  | TOC= | | | **10** |  |  |  | Sum= | **17** |

↑ ↑

|  |
| --- |
| When the sum of the durations of the critical activities is greater than the time of completion (TOC), then there are parallel paths in the critical path. |

**CPM: Crashing**

|  |
| --- |
| “Crashing” a project is to reduce the time of completion of the project. |

Consider the project.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** | **ES** | **EF** | **SA** | **T** | **LS** | **LF** | **Slk** |
| 1 | -- | 5 | 0 | 5 | 3 | 5 | 0 | 5 | 0 |
| 2 | -- | 2 | 0 | 2 | 4,5 | 2 | 2 | 4 | 2 |
| 3 | 1 | 2 | 5 | 7 | 6,7 | 2 | 5 | 7 | 0 |
| 4 | 2 | 3 | 2 | 5 | 6,7 | 3 | 4 | 7 | 2 |
| 5 | 2 | 3 | 2 | 5 | 7 | 3 | 6 | 9 | 4 |
| 6 | 3,4 | 3 | 7 | 10 | -- | 3 | 7 | 10 | 0 |
| 7 | 3,4,5 | 1 | 7 | 8 | -- | 1 | 9 | 10 | 2 |

Assume linear cost functions.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **A** | **T** | **Cost** | **Crash T** | **Crash Cost** | **Cost Rate** | **Slack** |
| 1 | 5 | 100 | 2 | 160 | 20 | 0 |
| 2 | 2 | 40 | 1 | 55 | 15 | 2 |
| 3 | 2 | 50 | 1 | 75 | 25 | 0 |
| 4 | 3 | 60 | 2 | 75 | 15 | 2 |
| 5 | 3 | 50 | 1 | 70 | 10 | 4 |
| 6 | 3 | 60 | 1 | 120 | 30 | 0 |
| 7 | 1 | 30 | 1 | 30 | 0 | 2 |
| TOC | 10 |  |  |  |  |  |
| Cost |  | 390 |  | 585 |  |  |

|  |
| --- |
| Objective: Crash project by 2. Crash from TOC=10 to TOC=8.  Crash Option 1: reduce activity 1 by 2.  Crash Option 2: reduce activity 6 by 2.  Crash Option 3: reduce activity 1 by 1 and activity 6 by 1. |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Option 1.** | | | **Option 2.** | | | **Option 3.** | | |
| **A** | **T** | **Cost** | **Slack** | **T** | **Cost** | **Slack** | **T** | **Cost** | **Slack** |
| 1 | **3** | 140 | 0 | 5 | 100 | 0 | **4** | 120 | 0 |
| 2 | 2 | 40 | 0 | 2 | 40 | 2 | 2 | 40 | 1 |
| 3 | 2 | 50 | 0 | 2 | 50 | 0 | 2 | 50 | 0 |
| 4 | 3 | 60 | 0 | 3 | 60 | 2 | 3 | 60 | 1 |
| 5 | 3 | 50 | 2 | 3 | 50 | 2 | 3 | 50 | 2 |
| 6 | 3 | 60 | 0 | **1** | 120 | 0 | **2** | 90 | 0 |
| 7 | 1 | 30 | 2 | 1 | 30 | 0 | 1 | 30 | 1 |
| TOC | 8 |  |  | 8 |  |  | 8 |  |  |
| Cost |  | 430 |  |  | 450 |  |  | 440 |  |

**CPM: Crashing with Parallel Paths**

|  |
| --- |
| Change duration of Activity “2” to 4  resulting in parallel paths in the critical path. |



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **A** | **PA** | **T** |  |  |
| 1 | -- | 5 |  |  |
| 2 | -- | 4 |  |  |
| 3 | 1 | 2 |  |  |
| 4 | 2 | 3 |  |  |
| 5 | 2 | 3 |  |  |
| 6 | 3,4 | 3 |  |  |
| 7 | 3,4,5 | 1 |  |  |

Assume linear cost functions.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **A** | **T** | **Cost** | **Crash T** | **Crash Cost** | **Cost Rate** | **Slack** |
| 1 | 5 | 100 | 2 | 160 | 20 | 0 |
| 2 | 4 | 10 | 1 | 55 | 15 | 0 |
| 3 | 2 | 50 | 1 | 75 | 25 | 0 |
| 4 | 3 | 60 | 2 | 80 | 20 | 0 |
| 5 | 3 | 50 | 1 | 70 | 10 | 2 |
| 6 | 3 | 60 | 1 | 120 | 30 | 0 |
| 7 | 1 | 30 | 1 | 60 | 0 | 2 |
| TOC | 10 |  |  |  |  |  |
| Cost |  | 360 |  | 585 |  |  |

|  |
| --- |
| Objective: Crash project by 1. Crash from TOC=10 to TOC=9.  Crash Option 1: Crash activity 2 by 1.  Crash Option 2: Crash activity 6 by 1. |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Option 1.** | | | **Option 2.** | | |
| **A** | **T** | **Cost** | **Slack** | **T** | **Cost** | **Slack** |
| 1 | 5 | 100 | 0 | 5 | 100 | 0 |
| 2 | **3** | 25 | 1 | 4 | 10 | 0 |
| 3 | 2 | 50 | 0 | 2 | 50 | 0 |
| 4 | 3 | 60 | 1 | 3 | 60 | 0 |
| 5 | 3 | 50 | 3 | 3 | 50 | 1 |
| 6 | 3 | 60 | 0 | **2** | 90 | 0 |
| 7 | 1 | 30 | 2 | 1 | 30 | 1 |
| TOC | 10 |  |  | 9 |  |  |
| Cost |  | 375 |  |  | 390 |  |

**Rules for Crashing a Network**

|  |
| --- |
| 1. Only crash critical activities.  2. Only crash one critical activity at a time.  3. Only crash a critical activity by one time period at a time.  4. When a critical activity is crashed, then critical activities in the network can become non-critical and non-critical activities in the network can become critical.  5. When a critical activity is crashed the network may not be crashed due to parallel paths.  6. Monitor crash results hierarchically. |

Consider the example.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Activity |  |  | Activity |  | Activity |  |
| A |  |  | B |  | C |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  | Activity |  |
|  |  |  |  |  | D |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A |  |  | B |  | C |  | Activity | A | B | C | D |  |  |
| 2 |  |  | 2 |  | 2 |  | Duration | 2 | 2 | 2 | 4 | TOC |  |
|  |  |  |  |  |  |  | Slack | 0 | 0 | 0 | 0 | 6 | Original |
|  |  |  |  |  | D |  | Slack | 0 | 0 | 0 | 0 | 5 | Crash activity A by 1 |
|  |  |  |  |  | 4 |  | Slack | 0 | 1 | 1 | 0 | 5 | Crash activity B by 1 |
|  |  |  |  |  |  |  | Slack | 0 | 0 | 0 | 0 | 4 | Crash activity D by 1 |
|  |  |  |  |  |  |  | Slack | 0 | 0 | 0 | 1 | 4 | Crash activity D by 1 |

Now Consider RFP Example with parallel paths in critical path.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Activity | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |  |
| Duration | 5 | 4 | 2 | 3 | 3 | 3 | 1 | TOC |  |
| Slack | 0 | 0 | 0 | 0 | 2 | 0 | 2 | 10 | Original |
|  |  |  |  |  |  |  |  |  |  |
| Activity | 1 | 2 | 3 | 4 | 5 | 6 | 7 | TOC |  |
| Duration | 5 | 4 | 2 | 3 | 3 | 2 | 1 |  | Crash Activity 6 by 1 |
| Slack | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 9 | Activities 5 and 7 slack reduced with TOC=9 |
| Duration | 4 | 4 | 2 | 3 | 3 | 2 | 1 |  | Crash Activity 1 by 1 |
| Slack | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 9 | Activity 3 becomes non-critical with TOC=9 |
| Duration | 4 | 4 | 2 | 3 | 3 | 1 | 1 |  | Crash Activity 6 by 1 |
| Slack | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 8 | Activities 5 and 7 become critical with TOC=8 |
| Duration | 4 | 4 | 2 | 2 | 3 | 1 | 1 |  | Crash activity 4 by 1 |
| Slack | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 8 | Activities 4 and 6 become non-critical with TOC=8 |

**CPM: Resource Leveling**

|  |
| --- |
| The reallocation of slack in activities to manage fluctuations in resource requirements. |

Initially, we consider the objective of the schedule to minimize time of completion (TOC). Resource leveling introduces additional objectives or constraints on variables such as labor, inventory, equipment, and cash availability.

Consider the following Gantt chart of the ‘Early Start Schedule’ where the number of parallel activities is being examined.

1. Early Start Schedule.

0 1 2 3 4 5 6 7 8 9 10

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |  | Slack |
| 1 | -- | 5 |  | 🡪 | 🡪 | 🡪 | 🡪 | 🡪 |  |  |  |  |  |  | 0 |
| 2 | -- | 2 |  | 🡪 | 🡪 |  |  |  |  |  |  |  |  |  | 2 |
| 3 | 1 | 2 |  |  |  |  |  |  | 🡪 | 🡪 |  |  |  |  | 0 |
| 4 | 2 | 3 |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  |  |  |  | 2 |
| 5 | 2 | 3 |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  |  |  |  | 4 |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | 🡪 | 🡪 | 🡪 |  | 0 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  | 🡪 |  |  |  | 2 |
|  | # |  |  | 2 | 2 | 3 | 3 | 3 | 1 | 1 | 2 | 1 | 1 |  |  |

Now consider other schedules through using slack.

1. Early Start Schedule.

0 1 2 3 4 5 6 7 8 9 10

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |  | Slack | Used | Left |
| 1 | -- | 5 |  | 🡪 | 🡪 | 🡪 | 🡪 | 🡪 |  |  |  |  |  |  | 0 | 0 | 0 |
| 2 | -- | 2 |  | 🡪 | 🡪 |  |  |  |  |  |  |  |  |  | 2 | 0 | 2 |
| 3 | 1 | 2 |  |  |  |  |  |  | 🡪 | 🡪 |  |  |  |  | 0 | 0 | 0 |
| 4 | 2 | 3 |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  |  |  |  | 2 | 0 | 2 |
| 5 | 2 | 3 |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  |  |  |  | 4 | 0 | 4 |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | 🡪 | 🡪 | 🡪 |  | 0 | 0 | 0 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  | 🡪 |  |  |  | 2 | 0 | 2 |
|  | # |  |  | 2 | 2 | 3 | 3 | 3 | 1 | 1 | 2 | 1 | 1 |  |  |  |  |

2. Late Start Schedule.

0 1 2 3 4 5 6 7 8 9 10

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |  | Slack | Used | Left |
| 1 | -- | 5 |  | 🡪 | 🡪 | 🡪 | 🡪 | 🡪 |  |  |  |  |  |  | 0 | 0 | 0 |
| 2 | -- | 2 |  |  |  | 🡪 | 🡪 |  |  |  |  |  |  |  | 2 | 2 | 0 |
| 3 | 1 | 2 |  |  |  |  |  |  | 🡪 | 🡪 |  |  |  |  | 0 | 0 | 0 |
| 4 | 2 | 3 |  |  |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  |  | 2 | 2 | 0 |
| 5 | 2 | 3 |  |  |  |  |  |  |  | 🡪 | 🡪 | 🡪 |  |  | 4 | 4 | 0 |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | 🡪 | 🡪 | 🡪 |  | 0 | 0 | 0 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  |  |  | 🡪 |  | 2 | 2 | 0 |
|  | # |  |  | 1 | 1 | 2 | 2 | 1 | 2 | 3 | 2 | 2 | 2 |  |  |  |  |

3. Level Activities Schedule.

0 1 2 3 4 5 6 7 8 9 10

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** | **T** |  |  |  |  |  |  |  |  |  |  |  |  | Slack | Used | Left |
| 1 | -- | 5 |  | 🡪 | 🡪 | 🡪 | 🡪 | 🡪 |  |  |  |  |  |  | 0 | 0 | 0 |
| 2 | -- | 2 |  | 🡪 | 🡪 |  |  |  |  |  |  |  |  |  | 2 | 0 | 2 |
| 3 | 1 | 2 |  |  |  |  |  |  | 🡪 | 🡪 |  |  |  |  | 0 | 0 | 0 |
| 4 | 2 | 3 |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  |  |  |  | 2 | 0 | 2 |
| 5 | 2 | 3 |  |  |  |  |  |  | 🡪 | 🡪 | 🡪 |  |  |  | 4 | 3 | 1 |
| 6 | 3,4 | 3 |  |  |  |  |  |  |  |  | 🡪 | 🡪 | 🡪 |  | 0 | 0 | 0 |
| 7 | 3,4,5 | 1 |  |  |  |  |  |  |  |  |  | 🡪 |  |  | 2 | 1 | 1 |
|  | # |  |  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |  |  |  |  |

**PERT: Program Evaluation and Review Technique**

|  |  |  |  |
| --- | --- | --- | --- |
| **A** | **PA** | **T** |  |
| 1 | -- | 5 |  |
| 2 | -- | 2 |  |
| 3 | 1 | 2 |  |
| 4 | 2 | 3 |  |
| 5 | 2 | 3 |  |
| 6 | 3,4 | 3 |  |
| 7 | 3,4,5 | 1 |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **A** | **t1=Optimistic** | **t2=Most Likely** | **t3=Pessimistic** | **E[Duration]**  **=(t1+4t2+t3)/6** | **V[Duration]**  **=[(t3–t1)/6]2** |
| 1 | 4.1 | 5 | 7.1 | 5.2 | 0.25 |
| 2 | 1.4 | 2 | 3.2 | 2.1 | 0.09 |
| 3 | 0.8 | 2 | 6.8 | 2.6 | 1.00 |
| 4 | 2.1 | 3 | 4.5 | 3.1 | 0.16 |
| 5 | 0.6 | 3 | 4.2 | 2.8 | 0.36 |
| 6 | 1.2 | 3 | 7.2 | 3.4 | 1.00 |
| 7 | 1 | 1 | 1 | 1 | 0 |

For example, for activity ‘1’: E[D] = ( 4.1 + 4(5) + 7.1 )/6 = 5.2

V[D] = [ ( 7.1 – 4.1 )/6 ]2  = 0.25

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **A** | **E[D]** | **V[D]** | **Slack** | **Critical Path** | **E[D]** | **V[D]** |
| 1 | 5.2 | 0.25 | 0 | Yes | 5.2 | 0.25 |
| 2 | 2.1 | 0.09 | 2.6 |  |  |  |
| 3 | 2.6 | 1.00 | 0 | Yes | 2.6 | 1.00 |
| 4 | 3.1 | 0.16 | 2.6 |  |  |  |
| 5 | 2.8 | 0.36 | 5.3 |  |  |  |
| 6 | 3.4 | 1.00 | 0 | Yes | 3.4 | 1.00 |
| 7 | 1 | 0 | 2.4 |  |  |  |
|  |  |  |  | Sum | 11.2 | 2.25 |
|  |  |  |  |  | E[TOC] | V[TOC] |

Assume TOC follows a normal distribution with mean 11.2 and variance 2.25 which implies standard deviation is 1.5=sqrt(2.25). Then, for a fixed value of T,

P[TOC < T]=P[Z < ( T – 11.2 )/1.5 ]= . Let Z be the standard normal random variate.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **T** | **Z** | **P[TOC<T]= ** |  | In Excel, use function, NORMSDIST(Z). |
| 10 | –0.8 | 0.212 |  | 0.2119=NORMSDIST(–0.8) |
| 11 | –0.133 | 0.447 |  | 0.4470=NORMSDIST(–0.133) |
| 12 | +0.533 | 0.703 |  | 0.7031=NORMSDIST(+0.533) |
| 13 | +1.2 | 0.885 |  | 0.8849=NORMSDIST(+1.2) |
| 14 | +1.867 | 0.969 |  | 0.9690=NORMSDIST(+1.867) |

**Normal Probability Distribution**

M.D. Harper, Ph.D.

Let the random variable, X, follow a ‘Normal Probability Distribution.’

Other terminology used to for the ‘Normal Probability Distribution’ includes:

Normal Distribution, Normal Density Function, Bell Curve, or Gaussian Distribution.

Consider the Normal graph:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | |  |
|  |  |  | |  |
| X  X X  X~N(  ,  | | | | |
| Where   = Probability  X = Random Variable  X = X1- = Random Variate   = Population Mean = E[X]  2 = Population Variance = V[X]   = Population Standard Deviation | | | Specifically,  P[ X < X ] =   P[ X < X1- ] = 1-  P[ X > X1- ] =   P[ X <  ] =  , thus, =X0.5  P[ X =  ] = and for any constant.  P[ X < X < X1- ] = 1-2  Empirical Rule:  P[ - < X < +] = 0.6826  P[ -2 < X < +2] = 0.9544  P[ -3 < X < +3] = 0.9972 | |

**Standard Normal Probability Distribution**

Now consider the transformation, Z = ( X -  ) /  , where E[Z]=0 and V[Z]=1.

Thus, Z ~ N( 0 , 1 ), called the Standard Normal Distribution. Consider the graphs:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | |  |
|  |  |  | |  |
| X  X X    X~N(  ,  | | | | |
|  |  |  | |  |
|  |  |  | |  |
| Z  Z Z  Z~N(  , =1  Z~N(  ,  | | | | |
| Where   = Probability  Z = Standard Normal Random Variable  Z = Z1- = Standard Normal Variate   = 0 = Population Mean  2 = 1 = Population Variance   = 1 = Population Standard Deviation | | | Specifically,  P[ Z < Z ] =   P[ Z < Z1- ] = 1-  P[ Z > Z1- ] =   P[ Z <  ] =  , thus, =X0.5  P[ X =  ] = and for any constant.  P[ Z < Z < Z1- ] = 1-2  Empirical Rule:  P[  < Z < ] = 0.6826  P[  < Z < ] = 0.9544  P[  < Z < ] = 0.9972  Determine values from:  **Standard Normal Probability Tables** | |

**Standard Normal Probability Table**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
| Z  Z  Z~N(  , 1 | | | |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Standard Normal Probability Table** | | | |  |  |  |
|  |  |  |  |  |  |  |
| Table represents the cumulative probability or Distribution Function. | | | | | | |
| For example, P[Z< (0.3+0.04)]=0.63307 | | | | | | |
|  |  |  |  |  |  |  |
| Z | 0 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
| 0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 |

P[ Z < 0 ] = P[ Z > 0 ] = 0.5

P[ Z < 1 ] = P[ Z > –1 ] = 0.84134

P[ Z > 1 ] = P[ Z < –1 ] = 1 – 0.84134 = 0.15866

P[ 0.2 < Z < 0.5 ] = P[ –0.5 < Z < –0.2 ] = 0.69146 – 0.57926 = 0.1122

P[ –0.2 < Z < 0.5 ] = P[ –0.5 < Z < 0.2 ] = 0.69146 + 0.57926 – 1 = 0.27072

P[ Z = 1.23 ] = 0

**Probabilities with PERT**

|  |
| --- |
| Consider the values from the PERT analysis of the project:  Let TOC be a normally distributed random variable   = E[TOC]=11.2 , Expected time of completion or Mean time of completion  2 = V[TOC]=2.25 , Variance of the time of completion   = SD[TOC]=1.5 , Standard Deviation of the time of completion = sqrt(Variance)  Also, Z=(TOC–E[TOC])/SD[TOC] = (TOC –  Thus, for a constant, K, P[TOC<=K]=P[Z<=(K–E[TOC])/SD[TOC]]=P[Z<(K– |

**Definitions.**

|  |
| --- |
| Since the expected time of completion is 11.2 days, probabilities will express the day of completion to be [11<=TOC<=12] or completed on the 12th day. Thus, the probability of completion at least one day early would be P[TOC<=11] and at least one day late P[TOC>=12]. However, we will interpret the statement “completed exactly one day or more ahead of schedule” to mean P[TOC<=10.2] which is P[TOC<=(11.2–1)]. Similarly, the statement “completed exactly one day or more behind schedule” would be P[TOC>=12.2]. Finally, it should be noted that mathematically for continuous distributions, the two expressions P[TOC<=11.2] and P[TOC<11.2] are equivalent. |

**Consider the questions.**

|  |
| --- |
| 1. What is the probability the project will be at least one day early?  P[TOC<11]=P[Z<(11–11.2)/1.5]=P[Z<–0.133]=0.4470  {Note: Probabilities such as 0.4470 can be obtained with the Excel function “=NORMSDIST(–0.1330)” which stands for the “Standard Normal Distribution”.}  2. What is the probability the project will be exactly one day or more behind schedule?  P[TOC>12.2]=P[Z>(12.2 –11.2)/1.5]=P[Z>+0.667]=1–0.7475=0.2525  3. What is the probability that the project will be completed within one day of its expected time of completion? Since ‘day of completion’ is [11<TOC<12], within one day would be one day ahead and behind expressed as [11–1<TOC<12+1]. Thus,  P[10<TOC<13]=P[(10–11.2)/1.5<Z<(13–11.2)/1.5]= P[–0.8<Z<+1.2]  =P[Z<+1.2] – P[Z<–0.8]=0.8849–0.2119=0.6731 |

**Random Variates with PERT**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| For a constant, K, consider the relations, P[TOC<=K]=P[Z<(K–= and P[Z<Z]=  Equating the random variates and solving yields, **K=  + Z \* **  Common Standard Normal Variates, Z , are   |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  | 0.9 | 0.95 | 0.975 | 0.99 | 0.8413 | 0.9332 | 0.9772 | 0.9938 | 0.9987 | | Z | 1.282 | 1.645 | 1.960 | 2.326 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |   Consider the problems.  1. What time of completion yields a 90% probability of meeting and a 10% probability of exceeding? TOC0.9= + Z \*  days  2. What is a 90% confidence interval of the time of completion?  (TOC0.05 , TOC0.95 )=(–  3. What confidence results in an interval of mean plus or minus two standard deviations?  Level of confidence =(2–1)=(2\*0.9772–1)=0.9544. A 95.44% confidence interval. |

**PERT with Parallel Paths**

|  |
| --- |
| Change Duration Estimates of Activity 5. |



|  |  |
| --- | --- |
| **A** | **PA** |
| 1 | -- |
| 2 | -- |
| 3 | 1 |
| 4 | 2 |
| 5 | 2 |
| 6 | 3,4 |
| 7 | 3,4,5 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **A** | **t1=Optimistic** | **t2=Most Likely** | **t3=Pessimistic** | **E[Duration]**  **=(t1+4t2+t3)/6** | **V[Duration]**  **=[(t3–t1)/6]2** |
| 1 | 4.1 | 5 | 7.1 | 5.2 | 0.25 |
| 2 | 1.4 | 2 | 3.2 | 2.1 | 0.09 |
| 3 | 0.8 | 2 | 6.8 | 2.6 | 1.00 |
| 4 | 2.1 | 3 | 4.5 | 3.1 | 0.16 |
| **5** | **5.2** | **7** | **15.4** | **8.1** | **2.89** |
| 6 | 1.2 | 3 | 7.2 | 3.4 | 1.00 |
| 7 | 1 | 1 | 1 | 1 | 0 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | **Critical Path** | | | |  |
|  |  |  |  | **Path 1-3-6** | | **Path 2-5-7** | |  |
| **A** | **E[D]** | **V[D]** | **Slack** | **E[D]** | **V[D]** | **E[D]** | **V[D]** |  |
| 1 | 5.2 | 0.25 | 0 | 5.2 | 0.25 |  |  |  |
| 2 | 2.1 | 0.09 | 0 |  |  | 2.1 | 0.09 |  |
| 3 | 2.6 | 1.00 | 0 | 2.6 | 1.00 |  |  |  |
| 4 | 3.1 | 0.16 | 2.6 |  |  |  |  | 🡨Non-Critical |
| 5 | 8.1 | 2.89 | 0 |  |  | 8.1 | 2.89 |  |
| 6 | 3.4 | 1.00 | 0 | 3.4 | 1.00 |  |  |  |
| 7 | 1 | 0 | 0 |  |  | 1 | 0 |  |
|  |  |  | Sum | 11.2 | 2.25 | 11.2 | 2.98 |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **T=Day:** | **9** | **10** | **11** | **11.2** | **12** | **13** | **14** |
| Z136 | -1.467 | -0.800 | -0.133 | 0 | 0.533 | 1.200 | 1.867 |
| P[TOC136 < T]= | 0.071 | 0.212 | 0.447 | 0.5 | 0.703 | 0.885 | 0.969 |
| Z257 | -1.274 | -0.695 | -0.116 | 0 | 0.463 | 1.043 | 1.622 |
| P[TOC257 < T]= | 0.101 | 0.243 | 0.454 | 0.5 | 0.678 | 0.851 | 0.948 |
| P[TOC < T]= | 0.007 | 0.052 | 0.203 | 0.25 | 0.477 | 0.753 | 0.918 |

**Non-critical Activities in PERT with Parallel Paths**

**Consider in more detail how the project can be delayed beyond TOC.**

|  |  |  |  |
| --- | --- | --- | --- |
| 1. All activities are stochastic except activity “7”.  2. All activities are critical except activity “4”.  3. Critical Path consists of activities 1,3,6, & 2,5,7.  4. TOC is based on the mean duration of the critical activities.  5. Assume probability of exceeding TOC due to path “1,3,6” is 0.5  6. Assume probability of exceeding TOC due to path “2,5,7” is 0.5  7. Since only activity “4” is non-critical, consider the paths:   |  |  |  | | --- | --- | --- | | **1-3**-6  **2-4**-6 | **1-3**-7  **2-4**-7 | 2-**5**-7  2-**4**-7 |   8. If the duration of activities “2” & “4” exceed “1” & “3”, the project exceeds TOC.  9. If the duration of activity “4” exceeds activity “5”, the project exceeds TOC.  10. Assume probability of not exceeding TOC due to “2,4” exceeding “1,3” is  P[T2+T4<(5.2+2.6)]=P[Z<( (5.2+2.6) – (2.1+3.1) )/sqrt(0.09+0.16) ) ] = 2 ≈ 1  11. Assume probability of not exceeding TOC due to “4” exceeding “5” is  P[T4<8.1]=P[Z<( (8.1–3.1)/sqrt(0.16) ) ] = 1 ≈ 1  12. Therefore, the probability of not exceeding TOC is (0.5\*0.5\*1\*2) = 0.25  **CAUTION: As variances increase for any activity or as the mean duration of non-critical activities increase, the probability will decrease.** |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **PA** |  |  |  |  |  |  |  | Consider Paths:  1-3-6  1-3-7  2-4-6  2-4-7  2-5-7 |
| 1 | -- |  |  |  |  |  |  |  |
| 2 | -- |  |  |  |  |  |  |  |
| 3 | 1 |  |  |  |  |  |  |  |
| 4 | 2 |  |  |  |  |  |  |  |
| 5 | 2 |  |  |  |  |  |  |  |
| 6 | 3,4 |  |  |  |  |  |  |  |
| 7 | 3,4,5 |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | **Critical Path** | | | |  |
|  |  |  |  | **Path 1-3-6** | | **Path 2-5-7** | |  |
| **A** | **E[D]** | **V[D]** | **Slack** | **E[D]** | **V[D]** | **E[D]** | **V[D]** |  |
| 1 | 5.2 | 0.25 | 0 | 5.2 | 0.25 |  |  |  |
| 2 | 2.1 | 0.09 | 0 |  |  | 2.1 | 0.09 |  |
| 3 | 2.6 | 1.00 | 0 | 2.6 | 1.00 |  |  |  |
| 4 | 3.1 | 0.16 | 2.6 |  |  |  |  | 🡨Non-Critical |
| 5 | 8.1 | 2.89 | 0 |  |  | 8.1 | 2.89 |  |
| 6 | 3.4 | 1.00 | 0 | 3.4 | 1.00 |  |  |  |
| 7 | 1 | 0 | 0 |  |  | 1 | 0 |  |
|  |  |  | Sum | 11.2 | 2.25 | 11.2 | 2.98 |  |
| Note: A complete analysis would include all conditions and paths. | | | | | | | | |

**PERT Analysis of Critical and Non-critical Paths**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **T=Day:** | **9** | **10** | **11** | **11.2** | **12** | **13** | **14** |
| Z136 | -1.467 | -0.800 | -0.133 | 0 | 0.533 | 1.200 | 1.867 |
| P[TOC136 < T]= | 0.071 | 0.212 | 0.447 | 0.5 | 0.703 | 0.885 | 0.969 |
| Z257 | -1.274 | -0.695 | -0.116 | 0 | 0.463 | 1.043 | 1.622 |
| P[TOC257 < T]= | 0.101 | 0.243 | 0.454 | 0.5 | 0.678 | 0.851 | 0.948 |
| P[TOC < T]= | 0.007 | 0.052 | 0.203 | 0.25 | 0.477 | 0.753 | 0.918 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| E[T] | 5.200 | 2.100 | 2.600 | 3.100 | 8.100 | 3.400 | 1.000 |
| V[T] | 0.250 | 0.090 | 1.000 | 0.160 | 2.890 | 1.000 | 0.000 |
|  |  |  |  |  |  |  |  |
| Path | 1,3,6 | 2,5,7 | 1,3,7 | 2,4,6 | 2,4,7 | =Path |  |
|  |  |  | 7<6 | 2,4<1,3 | 4<5 | =Condition | |
| E[TOC] | 11.2 | 11.200 |  |  |  |  |  |
| V[TOC] | 2.25 | 2.980 | 2.400 | 2.600 | 5.000 | =Mean |  |
| T | 9 | 9 | 1.000 | 1.500 | 3.050 | =Standard Deviation | |
| Z | -1.467 | -1.274 | 2.4 | 2.12289 | 2.86299 | =Z |  |
| P[TOC<T] | 0.071 | 0.101 | 0.991802 | 0.98312 | 0.9979 | 0.007 |  |
| P[TOC<T] | 0.071 | 0.101 |  |  |  | 0.00721 |  |
| \_ |  |  |  |  |  |  |  |

Expected Monetary Value (EMV)

**PERT with EMV.**

Consider a company is based on the project.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **t1** | **t2** | **t3** | **E[Duration]**  **=(t1+4t2+t3)/6** | **V[Duration]**  **=[(t3–t1)/6]2** |  | **From PERT Analysis** | | |
| 1 | 4.1 | 5 | 7.1 | 5.2 | 0.25 |  | T | Z | P[TOC<T] |
| 2 | 1.4 | 2 | 3.2 | 2.1 | 0.09 |  | 9.000 | -1.467 | 0.071 |
| 3 | 0.8 | 2 | 6.8 | 2.6 | 1.00 |  | 10.000 | -0.800 | 0.212 |
| 4 | 2.1 | 3 | 4.5 | 3.1 | 0.16 |  | 11.000 | -0.133 | 0.447 |
| 5 | 0.6 | 3 | 4.2 | 2.8 | 0.36 |  | 12.000 | 0.533 | 0.703 |
| 6 | 1.2 | 3 | 7.2 | 3.4 | 1.00 |  | 13.000 | 1.200 | 0.885 |
| 7 | 1 | 1 | 1 | 1 | 0 |  |  |  |  |

In responding to an RFP, a company is offered two options to include in its project proposal. The time of completion of the project is expected to be on the 12th day (i.e., completed between 11 and 12 days as measured on a continuous scale, specifically, P[11<=TOC<=12], since completing the project in 11.5 days would be on the 12th day ). The two options for bonus and penalty schedules are reported:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Option 1. | |  | Option 2. | |
| If completed | Payout |  | If completed | Payout |
|  |  |  |  |  |
| At least 2 days early | 375 |  | At least 2 days early | 275 |
| 1 day early | 300 |  | 1 day early | 250 |
| On time, Day 12 | 200 |  | On time, Day 12 | 200 |
| 1 day late | 0 |  | 1 day late | 175 |
| At least 2 days late | 0 |  | At least 2 days late | 25 |

The company conducted a PERT analysis on Excel and reported the following results.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| From PERT Analysis | | |  | Option 1. | |  | Option 2. | |  |
| Min | Max | Prob |  | $ | EMV |  | $ | EMV |  |
|  |  |  |  |  |  |  |  |  |  |
|  | 10 | 0.212 |  | 375 | 79.446 |  | 275 | 58.260 |  |
| 10 | 11 | 0.235 |  | 300 | 70.533 |  | 250 | 58.777 |  |
| 11 | 12 | 0.256 |  | 200 | 51.227 |  | 200 | 51.227 |  |
| 12 | 13 | 0.182 |  | 0 | 0.000 |  | 175 | 31.821 |  |
| 13 |  | 0.115 |  | 0 | 0.000 |  | 25 | 2.877 |  |
|  | Sum= | 1.000 |  | Total= | **201.205** |  | Total= | **202.962** |  |

Note 1. Which variables would create a basis for strategic analysis.

Note 2. Discuss characteristics that would support selection of variables to modify.

Note 2. Along with ‘risk’ consider ‘utility’ within the decision process.